

Applications of Computer Engineering and Mathematics to Archaeometry, History and Archaeology

Constantin Papaodysseus

*Professor, School of Electrical and Computer Engineering, National Technical University
of Athens*

Computer Engineering and Mathematics team:

Dr.Eng. P. Rousopoulos, Dr.Eng. D.Arabadjis, Dr.Eng. M. Panagopoulos, Ph.D.
St. F. Giannopoulos, Ph.D. St. S. Zannos, Dr. Eng. M. Exarhos, Ph.D. St. E.
Kalfa, Ph.D. St. K. Chalatsis, C. Triantafyllou

Epigraphologist - Archaeologist: Professor **Steve Tracy**

*Institute for Advanced Study, Princeton, School of Historical Studies,
Princeton, New Jersey, U.S.A.,*

Ms. Lena Papazoglou-Manioudaki, *National Archaeological Museum of
Athens*

Classicist: Professor **Christopher Blackwell**

Furman University, Department of Classics, Greenville, South Carolina, U.S.A.

Three original applications and associated results

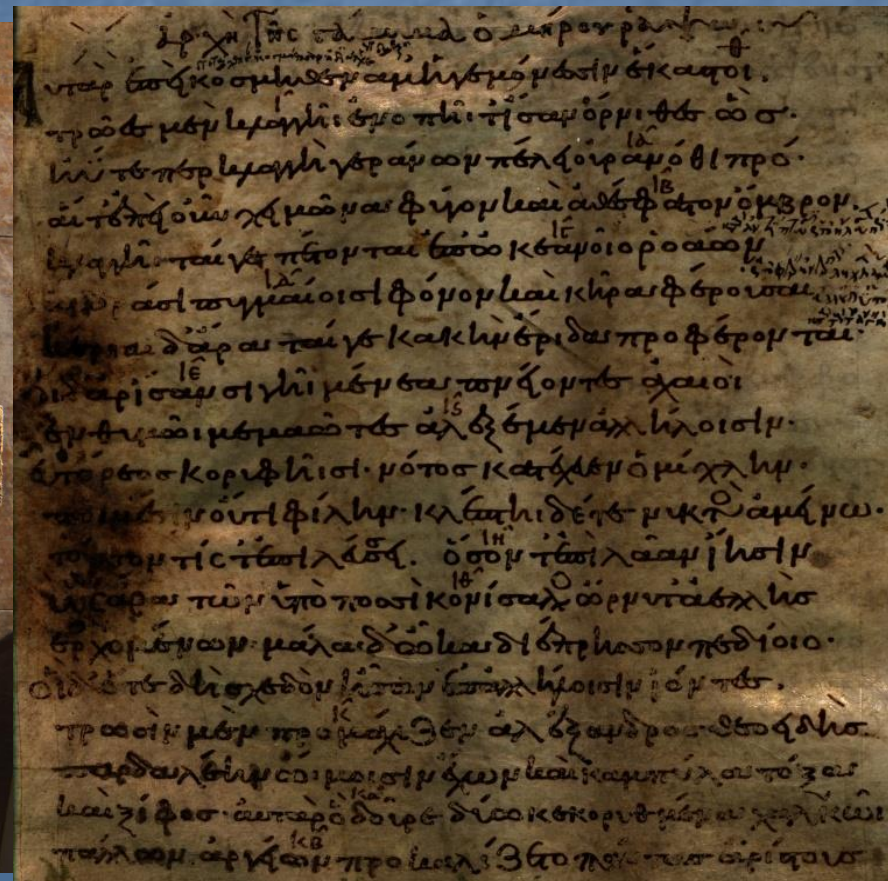
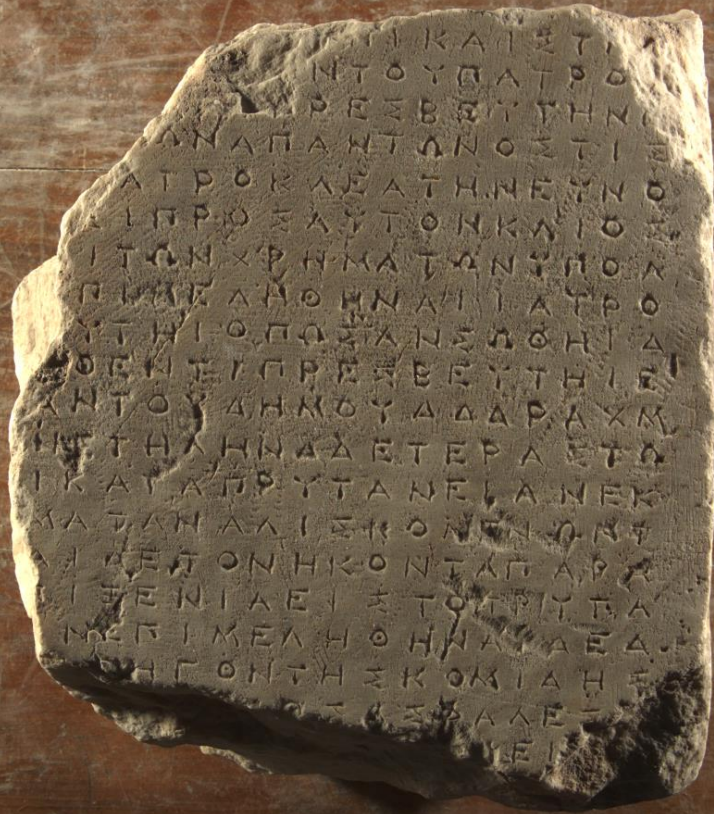
1. Automatic Identification of the Writer of Ancient Documents
2. Automated Reconstruction of Fragmented Archaeological Finds
3. Determination of the Method of Drawing of Celebrated Prehistoric Wall-Paintings

Automatic identification of the Writers of Ancient Greek Inscriptions, Byzantine Codices and Papyri

The scope of the research we present is the **automatic classification** of ancient manuscripts according to the writer (hand) that wrote or carved them.

A result of a successful identification is the **correct, objective dating** of the content of the manuscript

Digital images of an inscription and a Byzantine codex



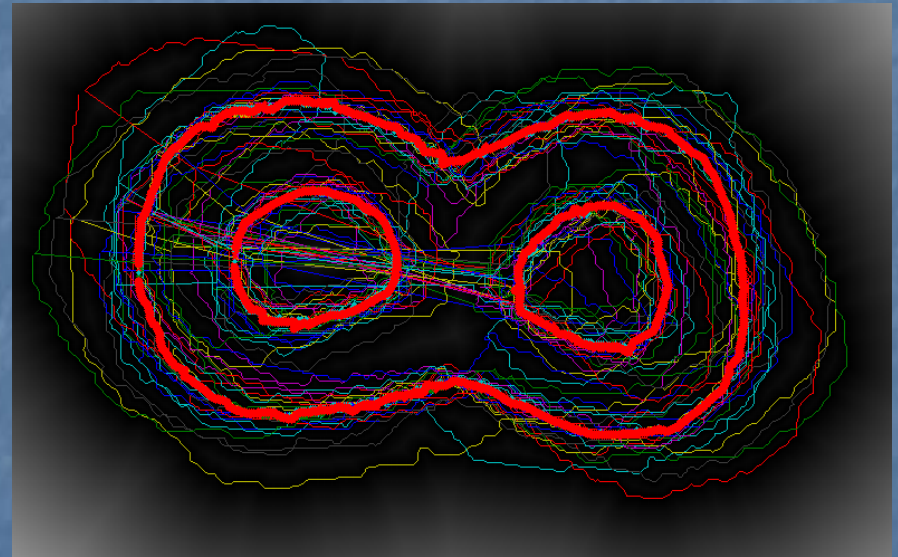
Three different methods of writer identification have been developed, to substantially increase the **degree of confidence** of the results.

First method

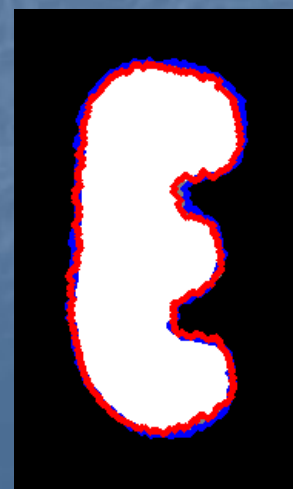
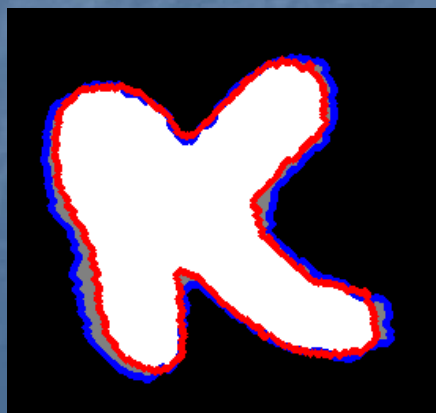
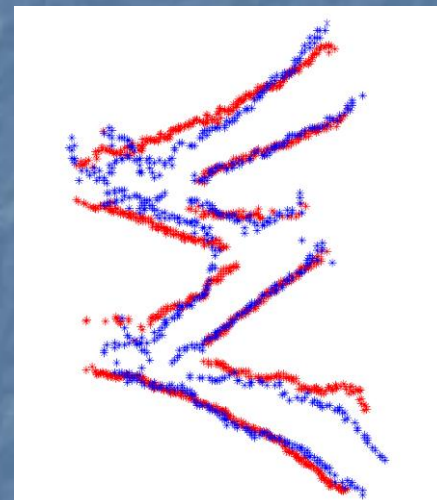
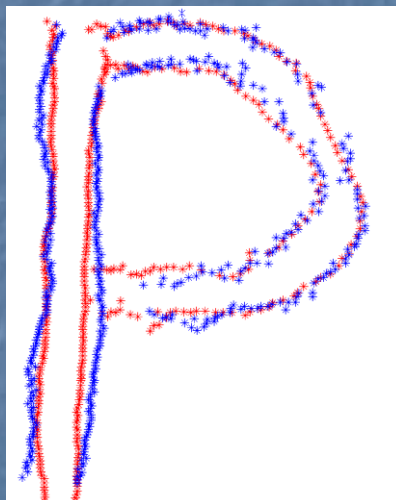
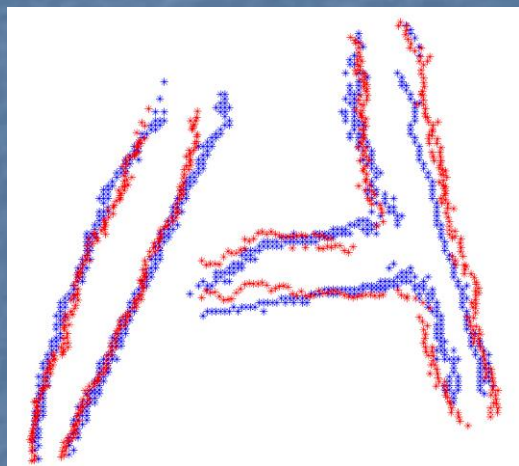
Basic concept: suppose that the writer has in his mind an **ideal prototype**, a kind of “**platonic**” **prototype**. Instead of this, the writer produces a **distorted, noisy** version of the letter.

Hence, a first approach towards writer identification is to suppress the noise, from different realizations of the same alphabet symbol and same writer.

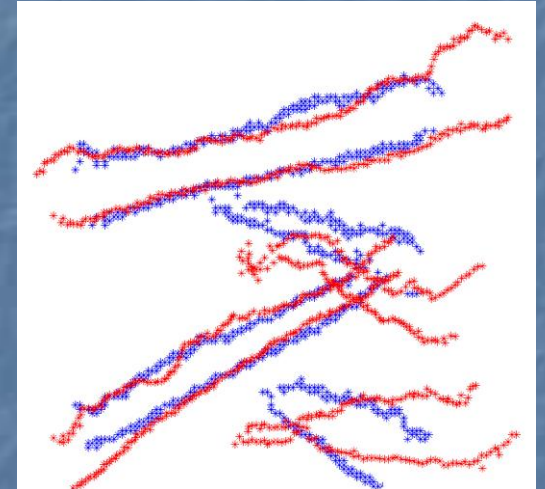
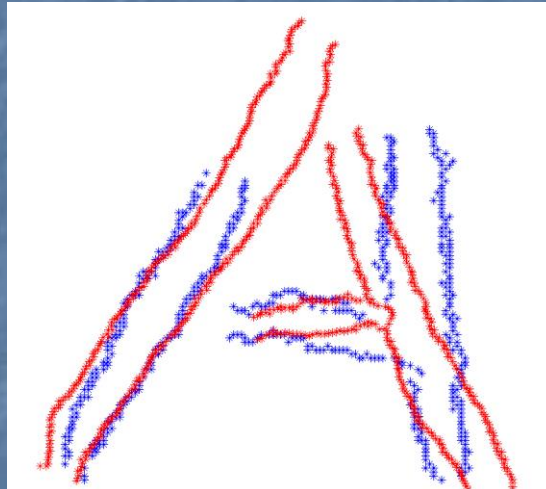
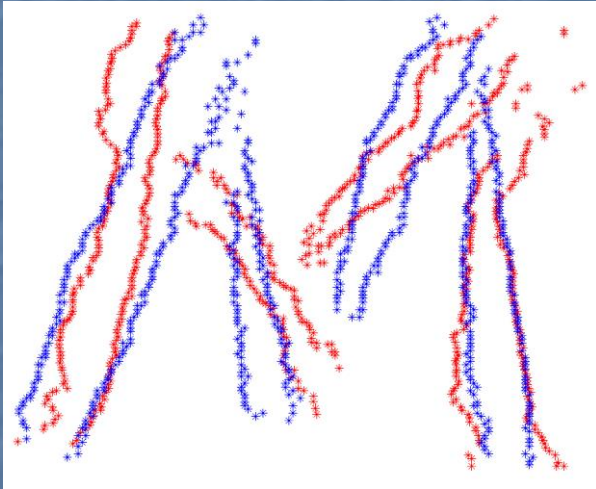
All letter contours are matched sequentially and estimation of their ideal representative is obtained



Ideal prototypes of letters produced by the same writer in different documents



Ideal prototypes of letters produced by different writers in different documents



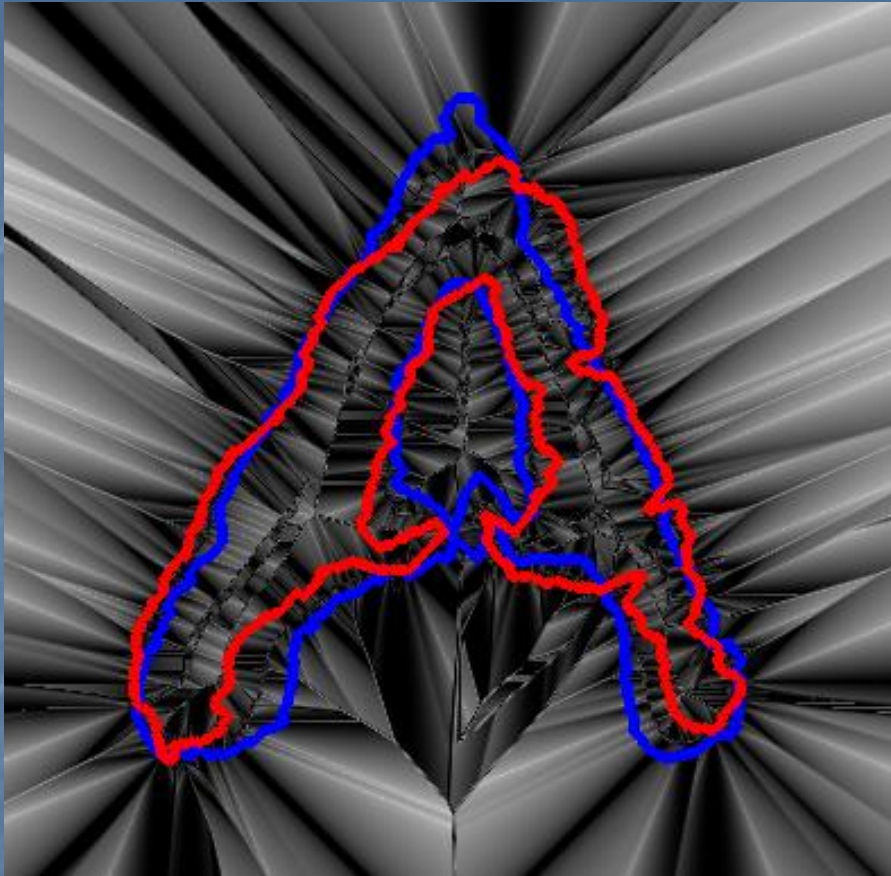
Second method

Basic concept: **Distortion** of each letter contour corresponds to tractable **deformation** of its **curvature**.

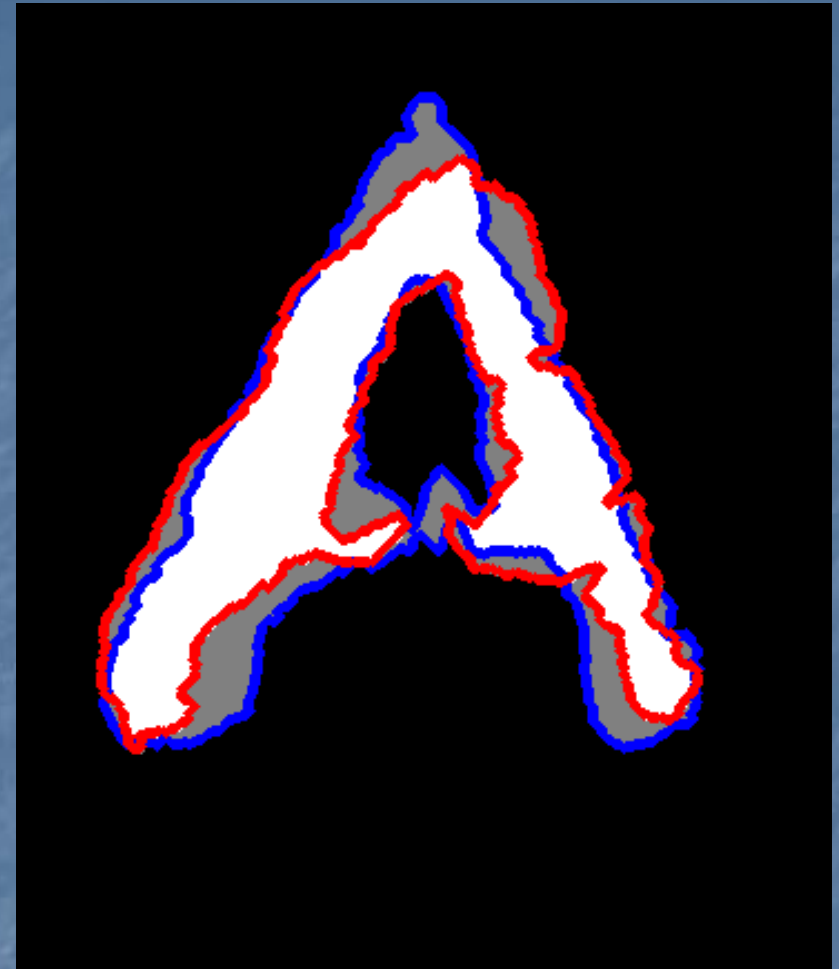
Letters' contours are fit so as to minimize point-to-point curvature differences.

The **similarity measure** combines:

- A) How close are the **contours' curvatures** and
- B) How small is the **area** of the optimally fit letters



Similarity measure A : Match of the contours' curvature skeletons



Similarity measure B : area enclosed by the 2 optimally matched contours

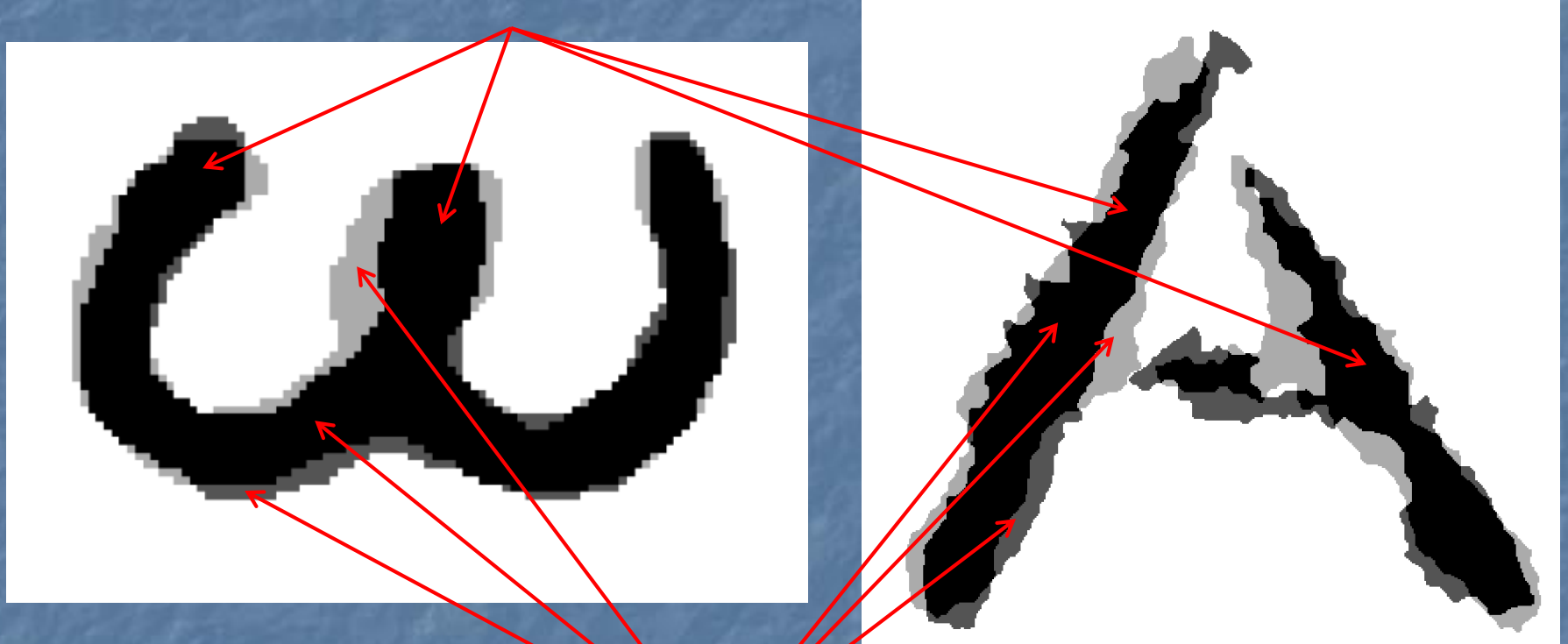
Third method

Basic concept: One can determine unambiguously optimal match between 2 different realizations of the same symbol if the normalized joint area of their shapes' binary representations is maximized.

$$\text{Matching Measure} = \frac{\text{Intersection of the matched letters}}{\text{Union of the matched letters}}$$

Optimal match of simply and multiply connected shapes

Letters' intersection



Letters' union

Application to a set of inscriptions

Prof. Tracy provided us with **46 inscriptions** to be classified according to the hand that made them.

He scrupulously abstained from disclosing any information concerning the identity of the inscription or of the writer that cut it or the age to which it is believed they belong or the method they were cut or the place they were found etc.

The goal was to assure that **any writer identification** result would be **based strictly on mathematics** and computer engineering terms.

Application to a set of inscriptions

Results :

Application of all three methods led to the same correct classification (100% accuracy) of the **46 inscriptions** into **10 writers** in accordance with the identification proposed by Prof. Tracy and other prominent epigraphists

Application to a set of Byzantine Codices

Prof. Chris Blackwell provided us with **23 byzantine codices** to be classified according to their writer. He also abstained from disclosing any information to us whatsoever.

Results :

All three methods offered correct classification (100% accuracy) of the **23 codices** into **4 writers**

Prof. Blackwell, after confirming the identification revealed that some of the codices belonged to *Venetus B* and *Escorialensis 3*

Related publications in approved international journals and conferences

- M. Panagopoulos, C. Papaodysseus, P. Roussopoulos, D. Dafi, S. Tracy "Automatic writer identification of ancient Greek inscriptions" *IEEE Pattern Analysis and Machine Intelligence*, vol. 31, no. 8, Aug 2009
- St. Tracy, C. Papaodysseus, P. Rousopoulos, M. Panagopoulos, D. Fragoulis, D. Dafi, Th. Panagopoulos. "Identifying Hands on Ancient Athenian Inscriptions: First Steps towards a Digital Approach". *Archaeometry* Vol. 49 Issue 4 Page 749 November 2007
- Stephen V. Tracy and Constantin Papaodysseus , "The study of Hands on Greek Inscriptions: The Need for a Digital Approach", *American Journal of Archaeology*, Volume 113, No 1, January 2009
- D. Arabadjis, F. Giannopoulos, C. Papaodysseus, S. Zannos, P. Rousopoulos, M. Panagopoulos, C. Blackwell, "New mathematical and algorithmic schemes for pattern classification with application to the identification of writers of important ancient documents", *Pattern Recognition*, vol. 46, pp. 2278-2296, 2013.
- C. Papaodysseus, P. Rousopoulos, F. Giannopoulos, S. Zannos, D. Arabadjis, M. Panagopoulos, C. Blackwell and S. Tracy, "Identifying the Writer of Ancient Inscriptions and Byzantine Codices. A novel approach", *Computer Vision and Image Understanding*, vol. 121, pp. 57-73, 2014.

Extended articles reporting the results of this research in international and Greek press

1. **The NewScientist**, "Computer reveals stone tablet 'handwriting' in a flash"
2. **Epoc Spectrum**, "Was Buchstaben in Stein verraten"
3. **Wiseenschaft**, "Handschriftenanalyse von gemeiselten Inschriften"
4. **Hbnews**, "Λογισμικό αναγνωρίζει το γραφικό χαρακτήρα σε λίθινες ελληνικές επιγραφές"
5. **BRIGHT SIDE OF NEWS**, "Computer attributes stone tablet writing to specific author"
6. **Η ΕΝΗΜΕΡΩΣΗ**, "CSI ΑρχαίαΑθήνα"
7. **in.gr**, "Λογισμικό αναγνωρίζει το γραφικό χαρακτήρα σε λίθινες ελληνικές επιγραφές"
8. **«ΤΑ ΝΕΑ»**, 'Αναγνώρισαν αρχαίες επιγραφές από τον γραφικό χαρακτήρα'
9. **«ΤΟ ΒΗΜΑ»**, 'Ηλεκτρονική ταυτοποίηση επιγραφών: Πρόγραμμα υπολογιστή μπορεί να ανακαλύπτει τον δημιουργό τους και να τις χρονολογεί με ακρίβεια "
10. **«Ελευθεροτυπία»**, 'Η ταυτότητα των αρχαίων γραφών'
11. **«ΤΟ ΒΗΜΑ»**, "Έλληνες επιστήμονες αναγνωρίζουν γραφείς αρχαίων επιγραφών"

Automated Reconstruction of Fragmented Objects – Application to the Reassembly of Celebrated Wall-Paintings

The development of an integrated information system that may assist in the reassembly of fragmented archaeological finds is of essential importance for the preservation of Cultural Heritage.

Application Case: The automatic reassembly of fragmented prehistoric wall paintings of Tyrins, Thera and Mycenae.

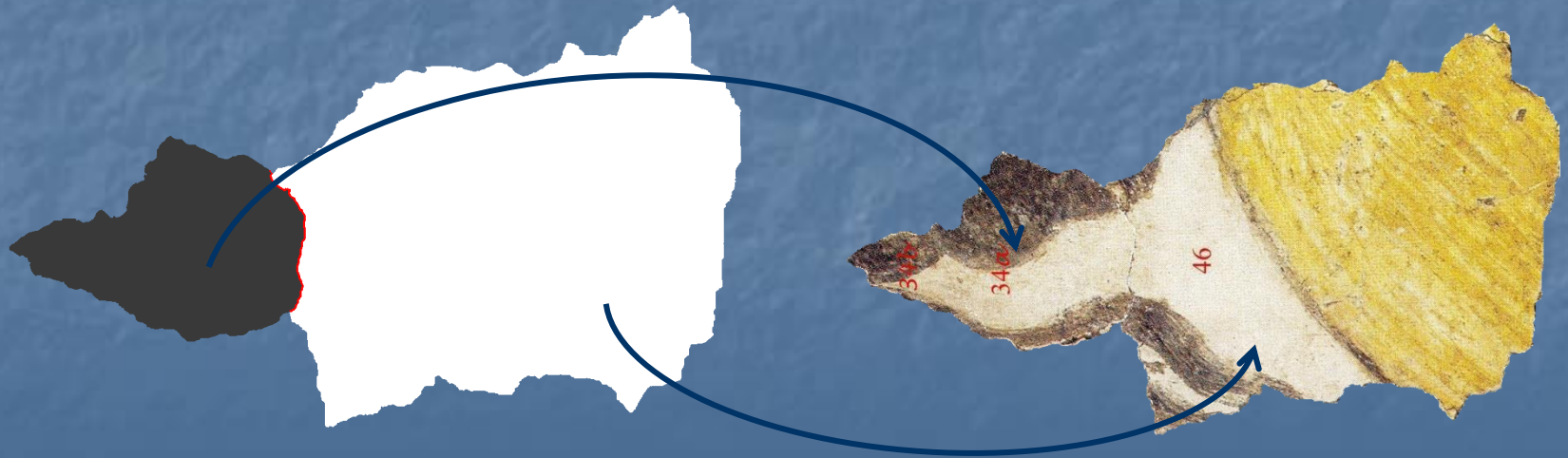
Automated Reconstruction

The developed system takes into account various aspects regarding the unearthed fragments :

1. Matching of the contours of the painted/plain sides of adjacent fragments
2. Three dimensional (3D) matching of the lateral surfaces of adjacent fragments
3. Continuation of colour and/or of the thematic content between the painted sides of matching fragments

I. Two-Dimensional Automated Reconstruction

The first subsystem takes into account only the two-dimensional geometric matching of the plain sides contours.



Application of the method to Akrotiri, Thera wall-paintings

The method has been applied to the prehistoric (c. 1650 B.C.) wall-paintings excavated in thousands of fragments at Akrotiri, Thera, Greece.

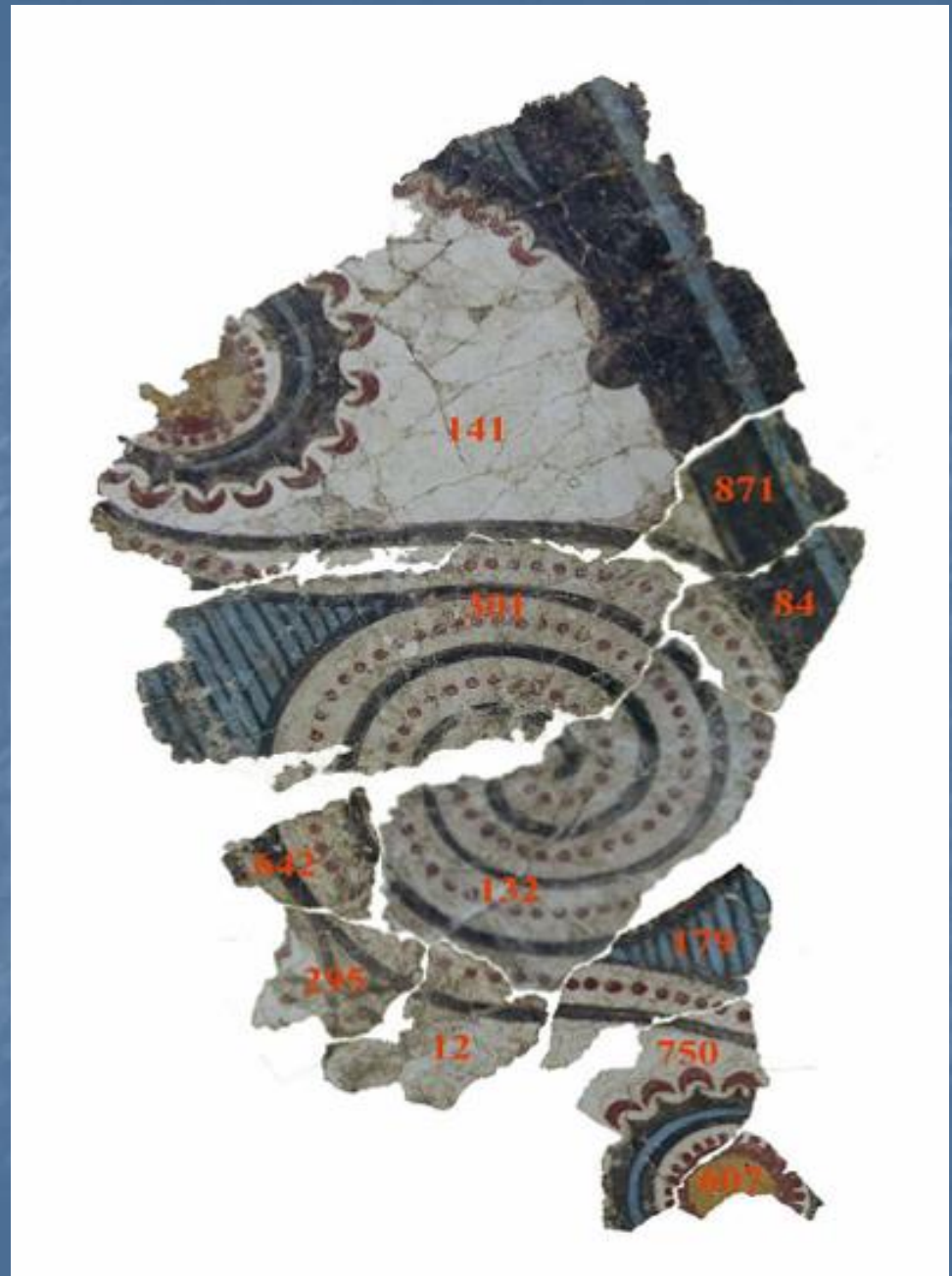
The authors have photographed the painted side of the wall-painting fragments and the developed system offered numerous matches between them for the first time.

Images of wall-painting fragments excavated at Akrotiri (Thera) shot by the authors



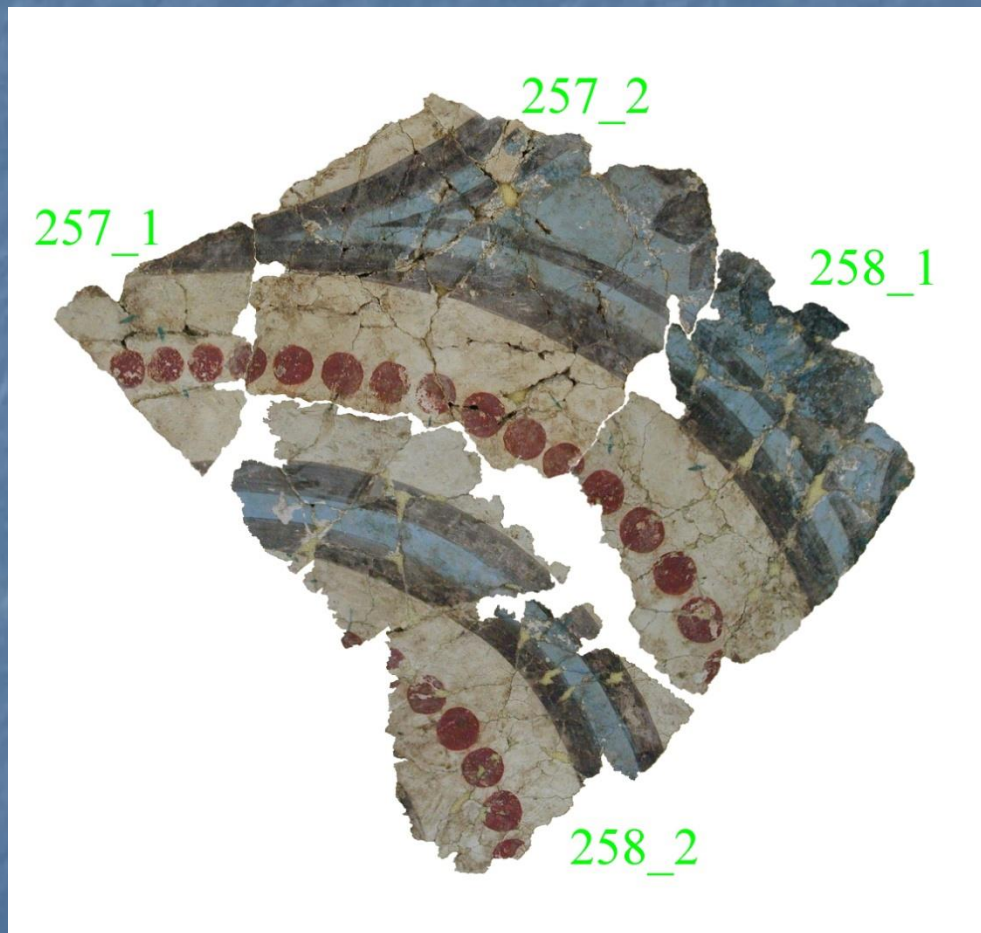
Reconstruction Results

Example of wall-painting parts automatically reconstructed by the 2D system for the first time



Reconstruction Results

Example of wall-
painting parts
automatically
reconstructed by
the 2D system for
the first time



Reconstruction Results

Example of wall-
painting parts
automatically
reconstructed by
the 2D system for
the first time



II. Three Dimensional Automated Reconstruction

The necessity of exploiting the 3D representation of the fragments:

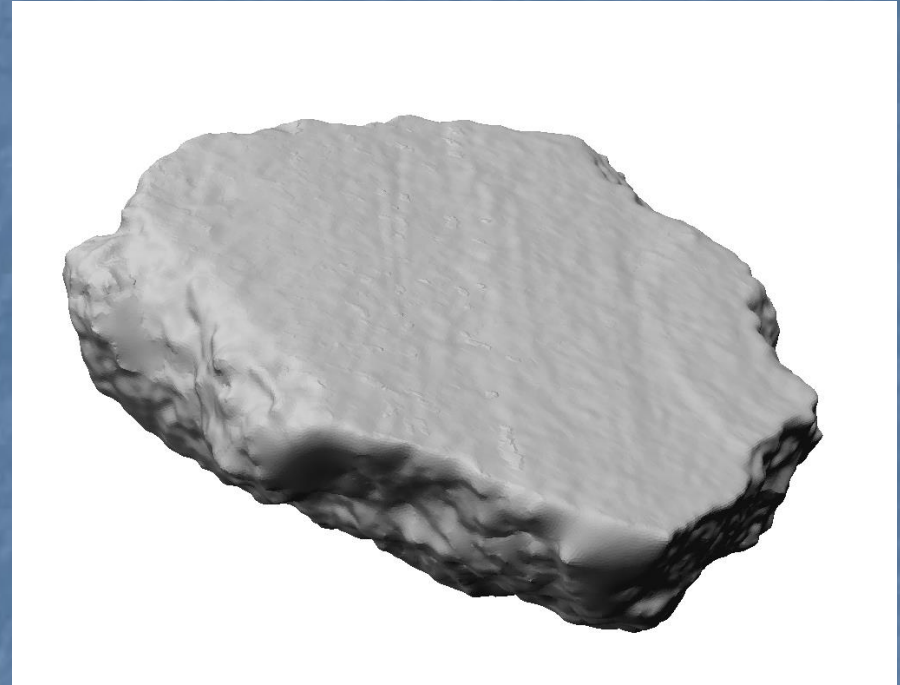
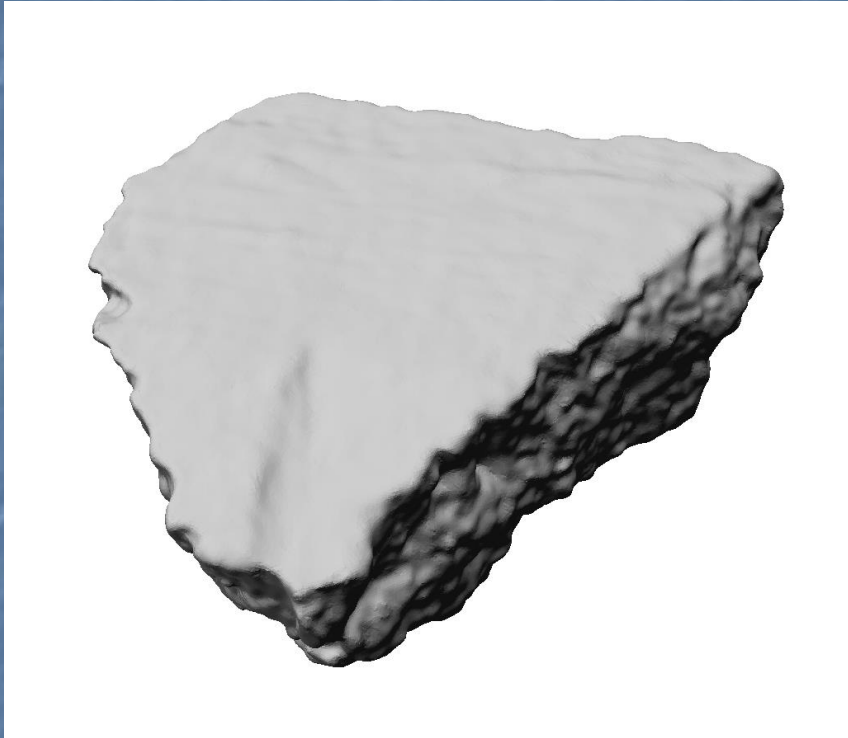
The 2D subsystem spots the existing matching between fragments, but often gives erroneous matches. We have attributed this to the limited information obtained in 2D, in connection with the large number of fragments and the possible small matching length.

II. Three Dimensional Automated Reconstruction

We have developed a subsystem that finds the actual matches between Fragments in three dimensions, using principles of 3D-Geometry, Calculus of Variations and 3D pattern recognition.

We have applied this subsystem to the virtual reassembly of islands of fragments excavated at Tyrins, in cooperation with Ms. Lena Papazoglou, ex director of the Prehistoric Department of the National Archaeological Museum of Greece.

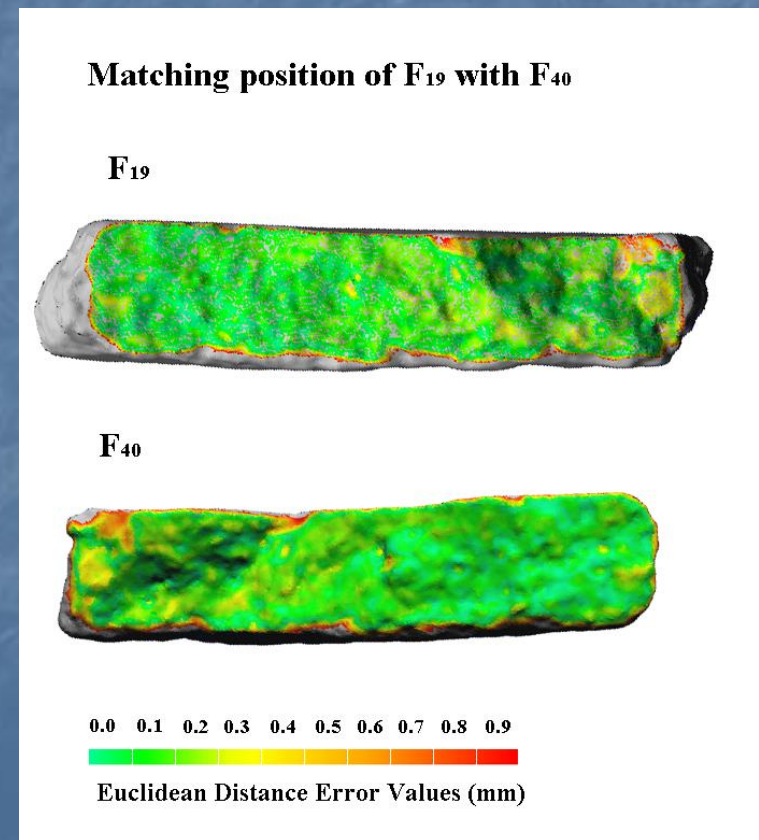
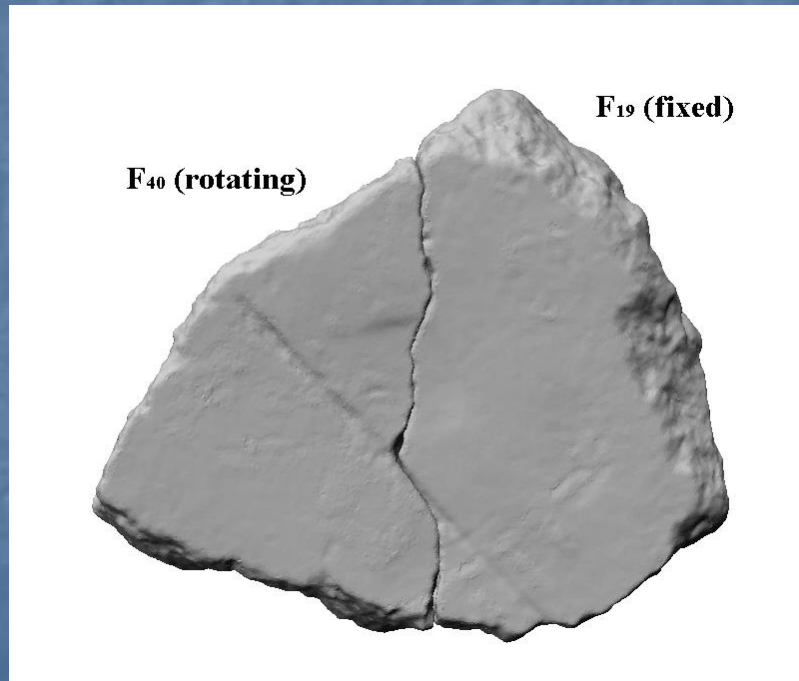
II. Three Dimensional Automated Reconstruction



Examples of 3D representation of fragments
obtained by the authors via 3D-scanner.

II. Three Dimensional Automated Reconstruction

Two fragments optimally matching in 3 dimensions; the matching position has been automatically suggested by the system.



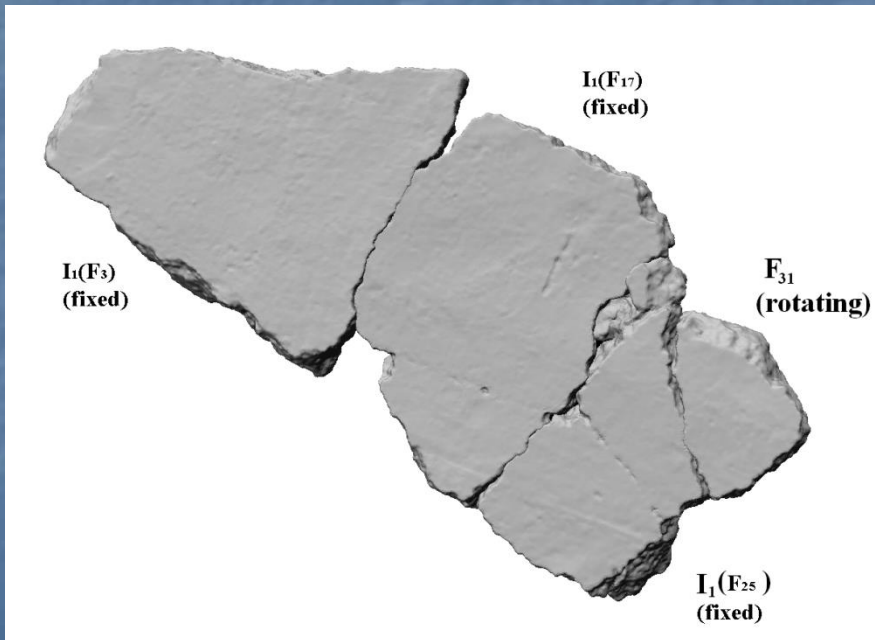
Application of the method

We have applied the aforementioned methodology to 41 fragments of prehistoric wall-paintings (14th and 13th century B.C.) from Tyrins and Mycenae kept in National Archaeological Museum of Greece.

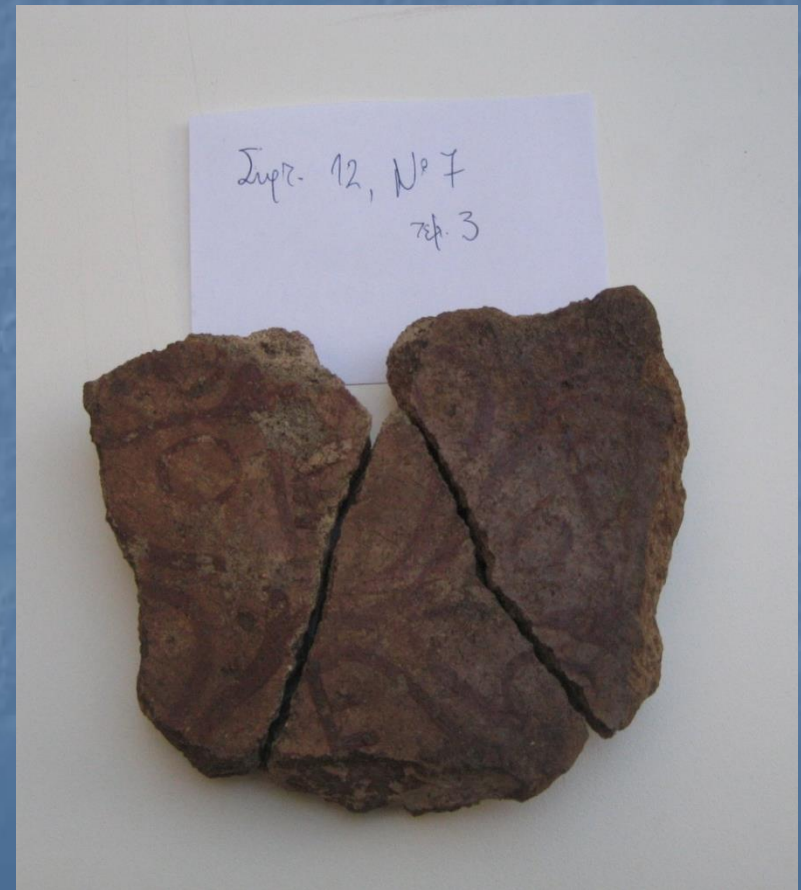
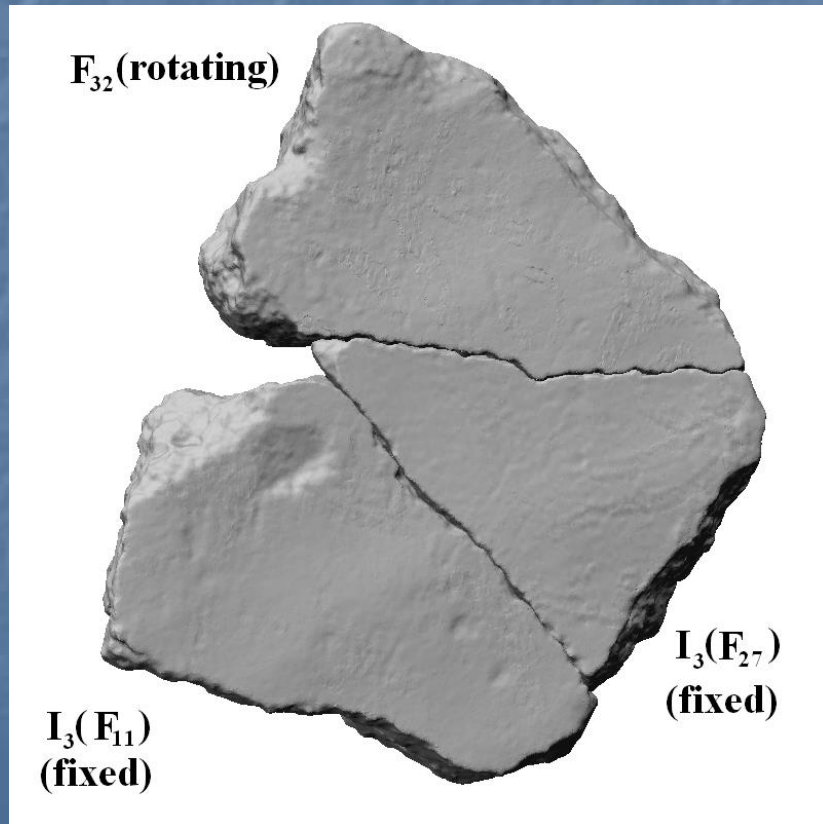
The developed system offered 9 islands of matching fragments consisting of 2-5 pieces.

These results comply with the manual reconstruction performed by Museum's scholars.

Reconstruction Results



Reconstruction Results



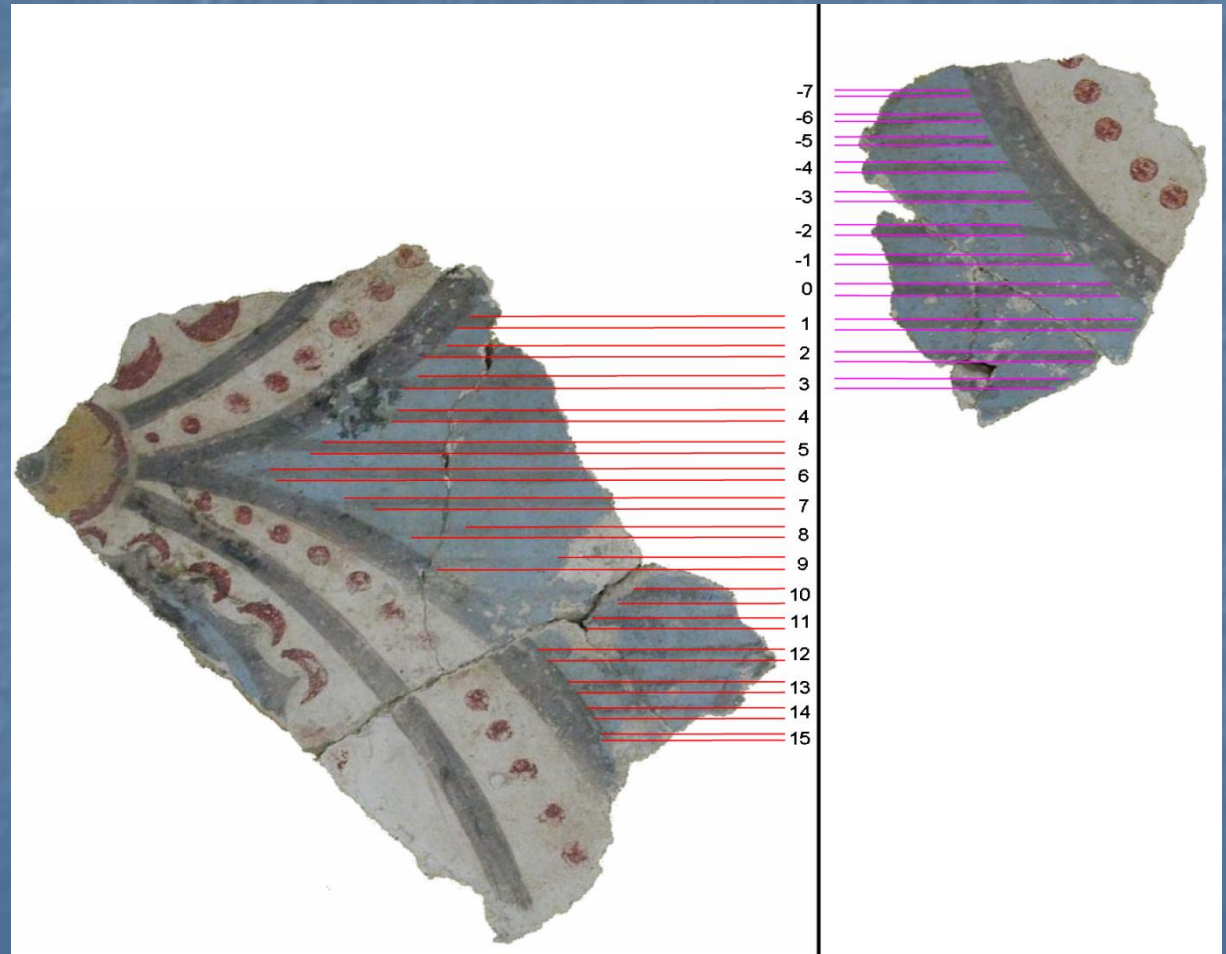
III. Exploitation of the thematic continuation

- We have developed a third subsystem that achieves optimal matching between fragments that are not necessarily in contact, by exploitation of the thematic content on the fragments' painted sides.
- We exploit the fact that the same geometric scheme may appear on different fragments belonging to the same theme.

III. Exploitation of the thematic continuation

Characteristic example :

We note that it is intrinsically very difficult for the dedicated personnel to spot this matching.



Characteristic example :

Matching of fragments in
loose contact following
Archimedes spiral
pattern



Related publications in international scientific journals

- Papaodysseus, C. Panagopoulos, T. Exarhos, M. Triantafillou, C. Fragoulis, D. Doumas, C., "Contour-shape based reconstruction of fragmented, 1600 BC wallpaintings", Signal Processing, IEEE Transactions on, vol 50, no. 6, pp. 1277-1288, 2002.
- C. Papaodysseus, D. Arabadjis, M. Exarhos, P. Rousopoulos, S. Zannos, M. Panagopoulos, L. Papazoglou-Manioudaki, "Efficient solution to the 3D problem of automatic wall-painting reassembly", Computers and Mathematics with Applications, vol. 64, pp. 2712-2734, 2012.
- D. Arabadjis, C. Papaodysseus, P. Rousopoulos, M. Panagopoulos, "On the Mathematical Formulation of the Problem of Reassembling Fragmented Objects: Two New Theorems ", Journal of Applied Mathematics and Computing, Springer, vol. 34, pp. 81-100, 2010.
- P. Rousopoulos, C. Papaodysseus, D. Arabatzis, M. Exarhos, M. Panagopoulos, "Reconstruction of c.1650 B.C. Fragmented Wall Paintings by Exploitation of the Thematic Content", International Journal of Imaging and Robotics, vol. 5, no. 11, 2011.

Determination of the Method of Drawing of Celebrated Prehistoric Wall-Paintings by means of Computer Engineering and Mathematics

Main Result of the Study : Celebrated prehistoric wall-paintings were drawn using geometric stencils

Examples of spiral themes we have studied



Spiral Geometric Prototypes

We have concluded that **all spiral themes**, initially decorating the upper floor of the prehistoric building “Xeste 3”, optimally correspond to **a single Archimedes spiral**.

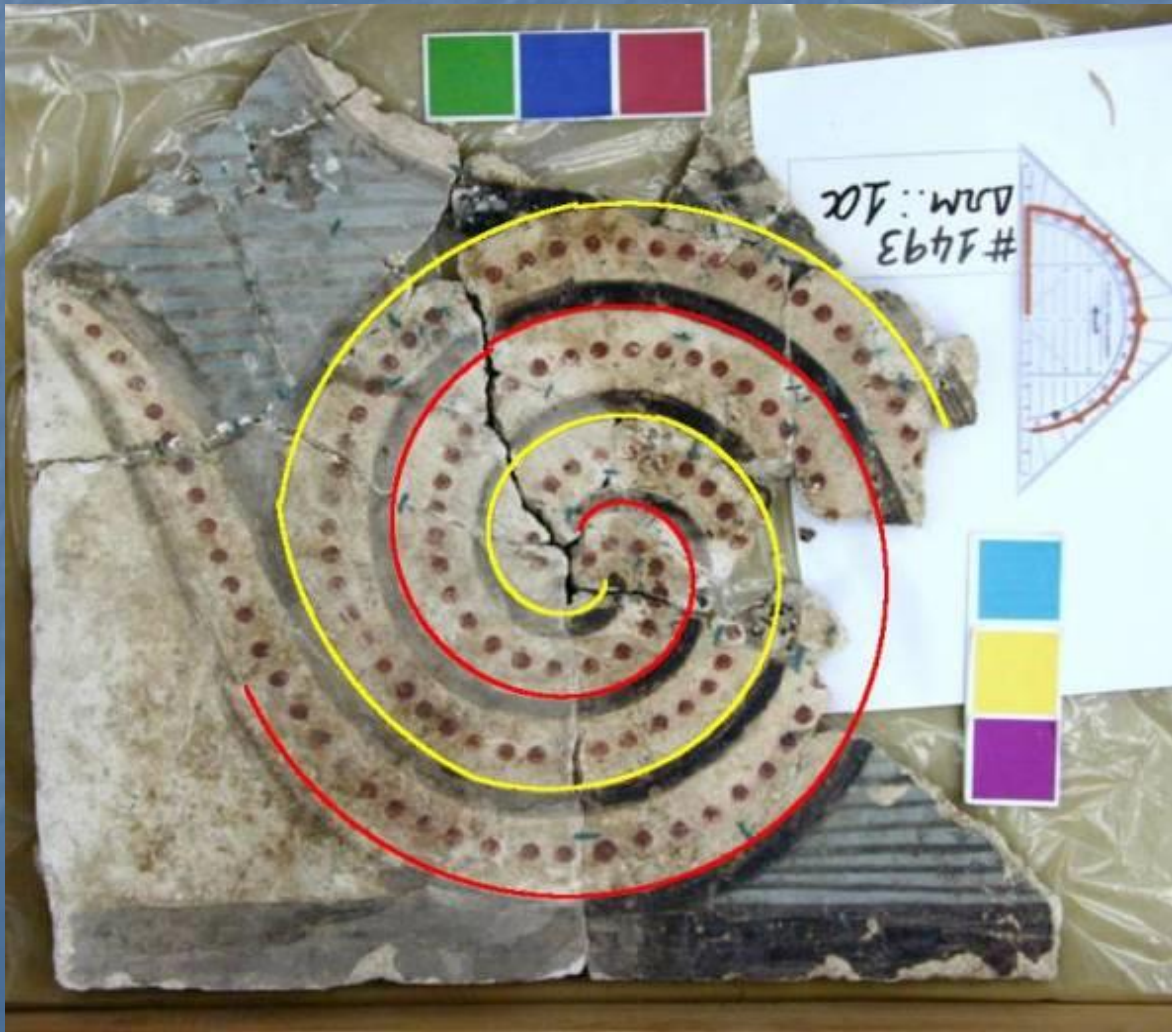
Spotted Archimedes Spiral Prototype

A **single linear spiral** obeying the equations

$$x = 0.159 \theta \cos \theta, y = 0.159 \theta \sin \theta$$

fits all **15 middle size drawn spirals**, in an excellent piece-wise manner, with an **average** error of less than **1/3 mm** and a **maximum** error of less than **0.8 mm**

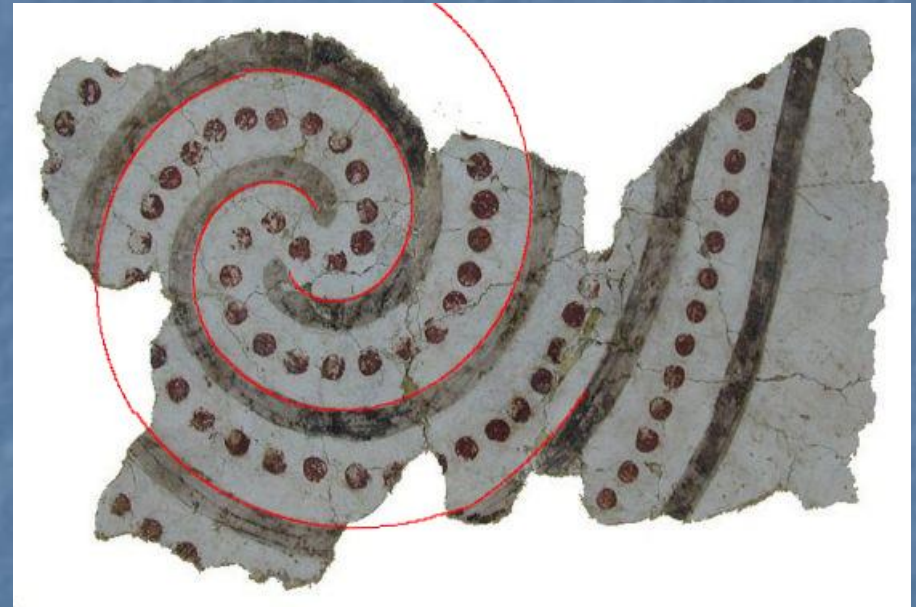
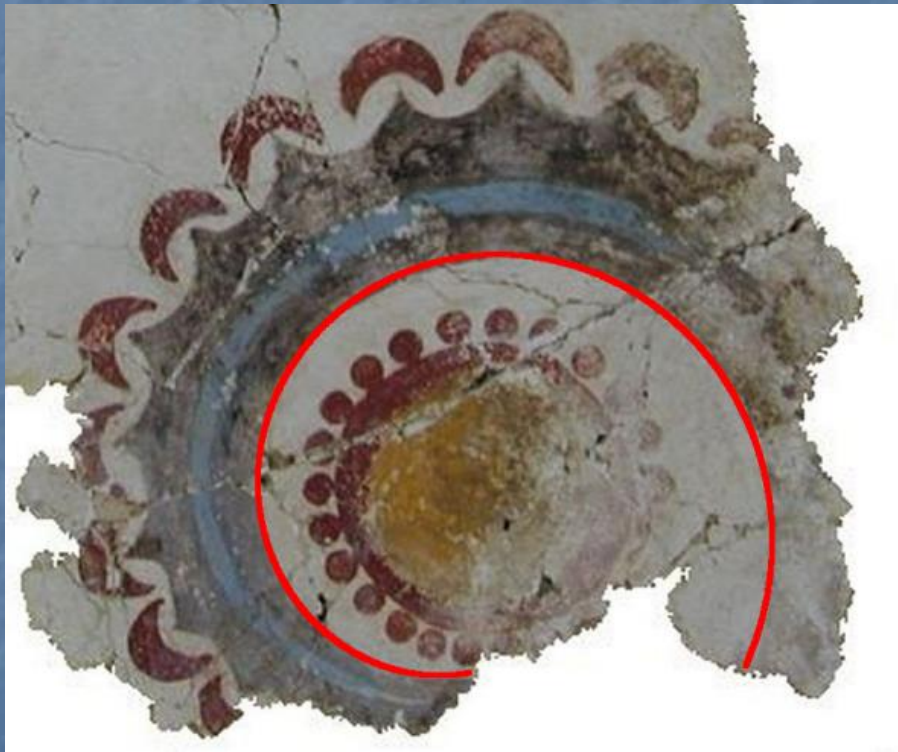
A linear spiral realization might appear c. 1350 years before its mathematic definition and treatment by Archimedes and Konon



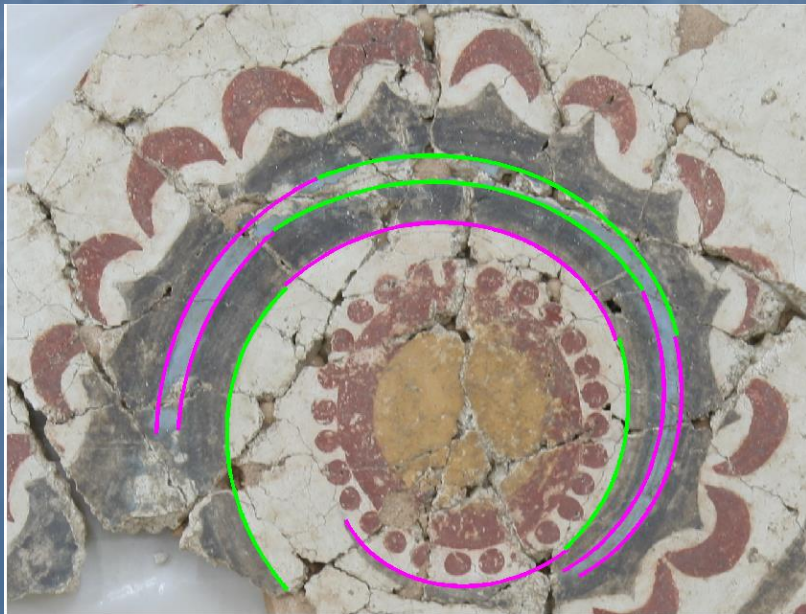
Further examples of excellent fitting of the determined Archimedes spiral prototype on the drawn spirals contour



Further examples of excellent fitting of the determined Archimedes spiral prototype on the drawn spirals contour



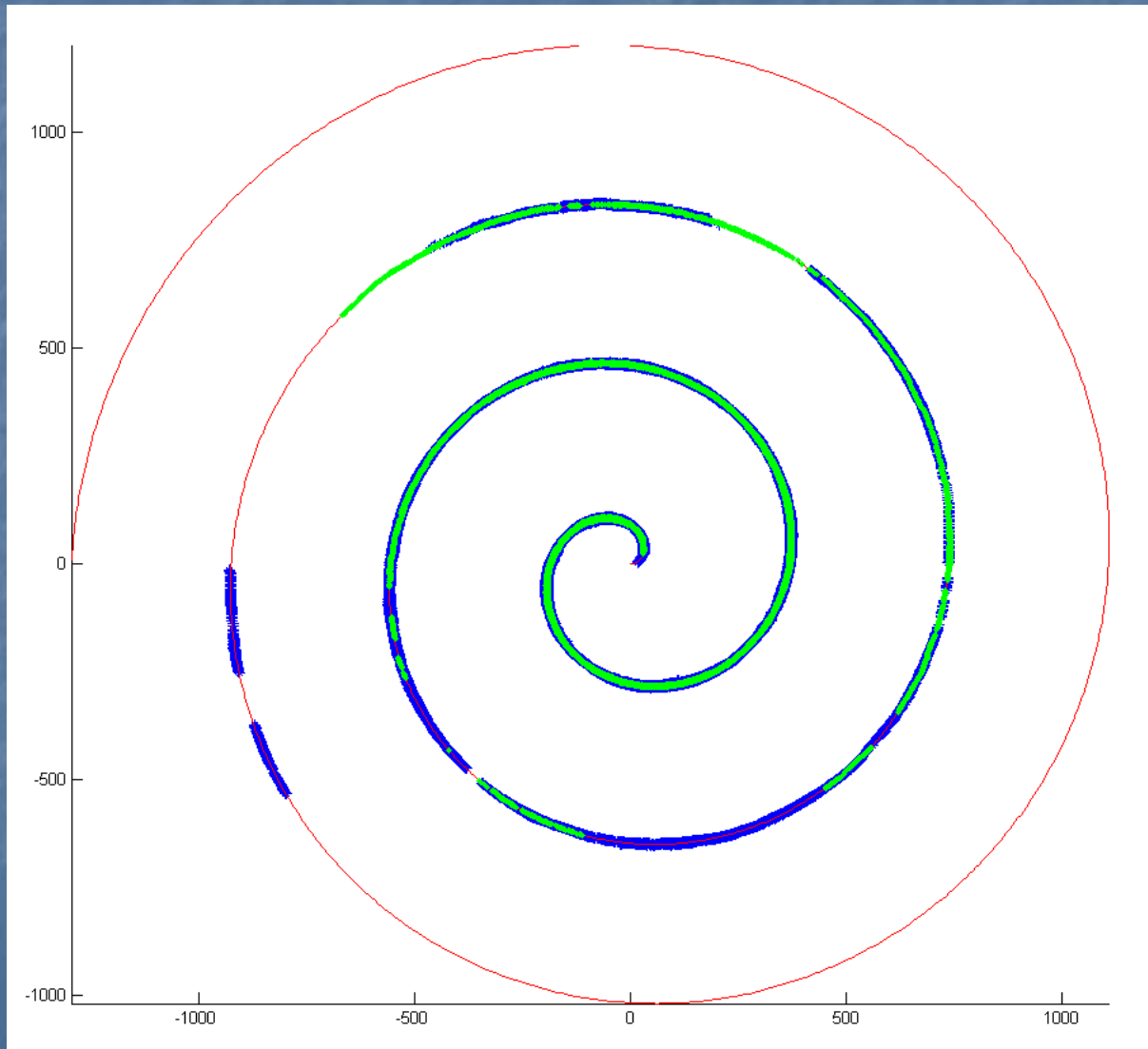
Spotted one-stroke
linear spiral parts
accurately generating
the inner and outer
outline of the wall-
painting spiral
themes.



Spotted one-stroke
linear spiral parts
accurately generating
the inner and outer
outline of the wall-
painting spiral
themes.



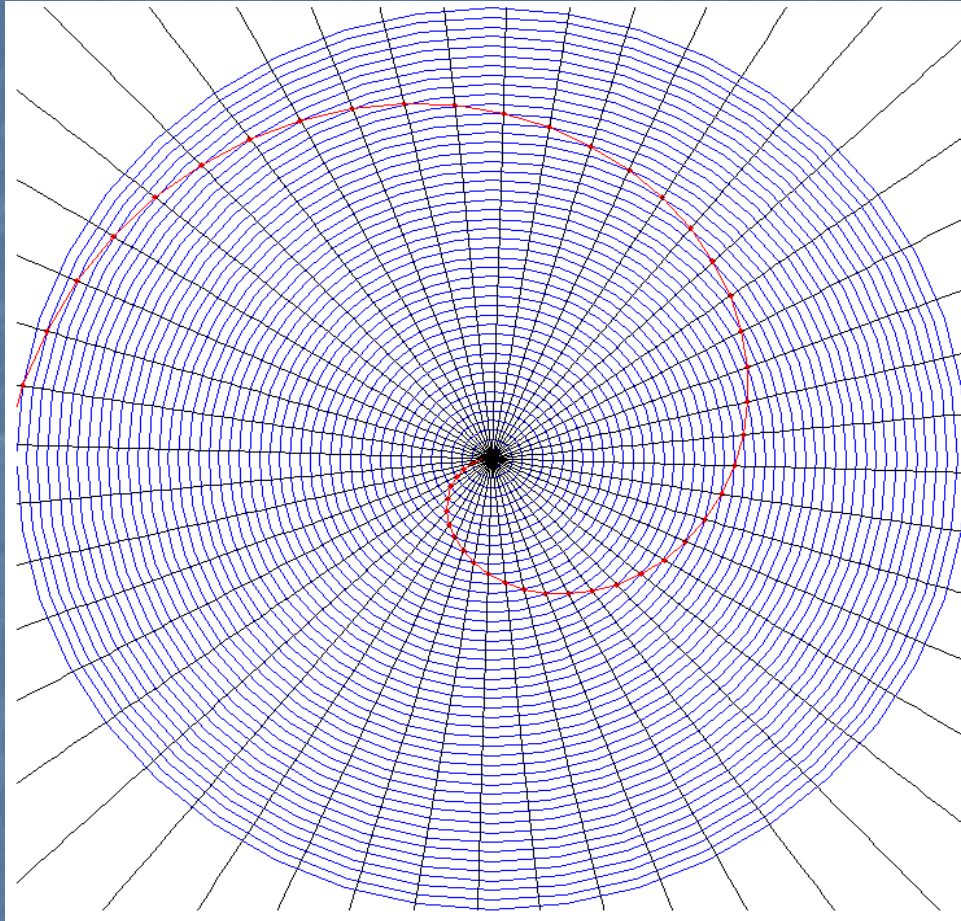
Outer(blue) and Inner(green) Outlines – to – Archimedes Spiral prototype correspondence



Geometric prototypes

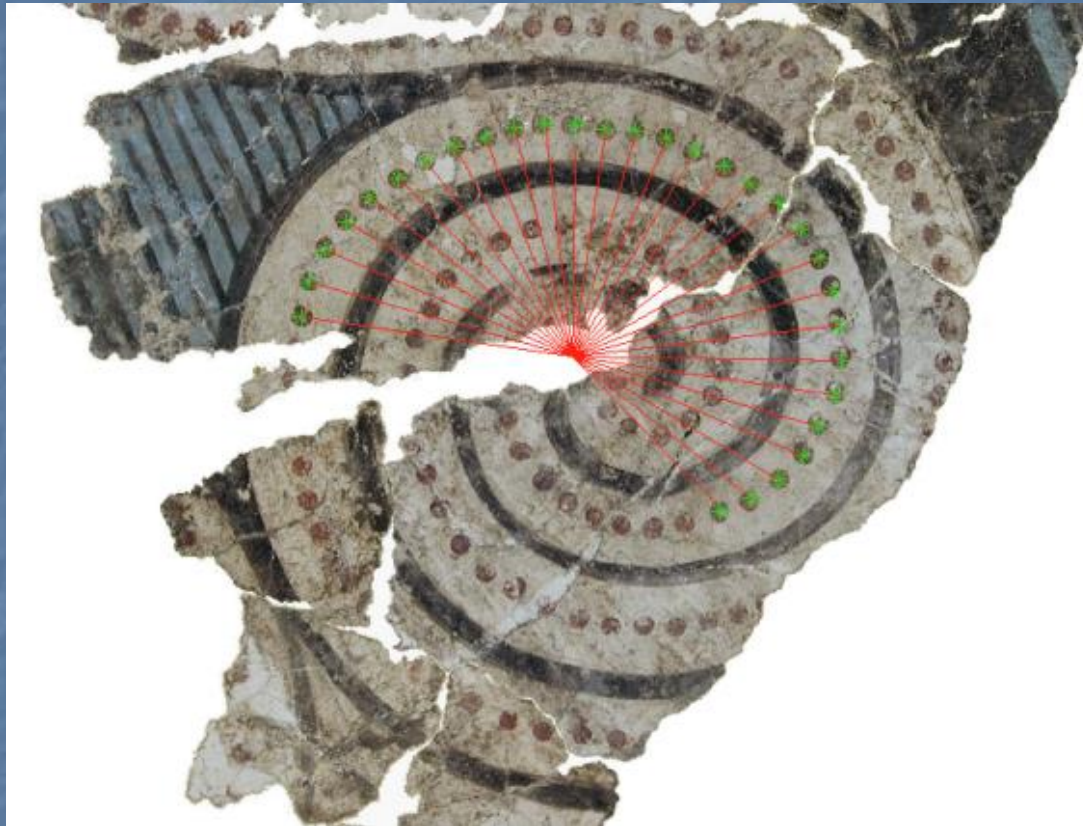
- The exact method of drawing these wall-paintings is, so far, unknown. However, a plausible method, consistent with the technological means of the era, may be the following.

Geometric prototypes



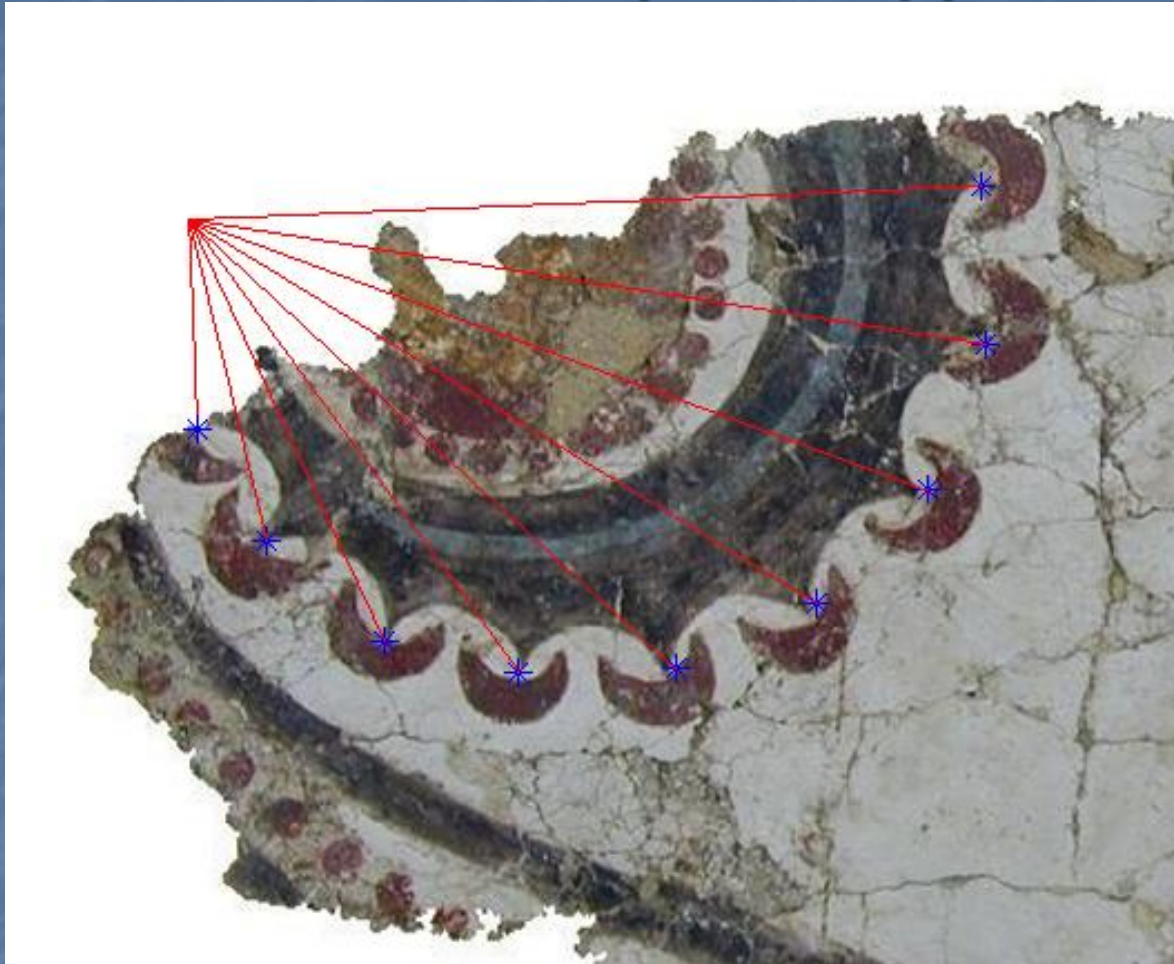
The circles are equidistant and the concurrent straight lines divide the entire 360° angle into N equal angular intervals.

Geometric prototypes



Decorative elements lying on radii of regular 48-gon.

Geometric prototypes

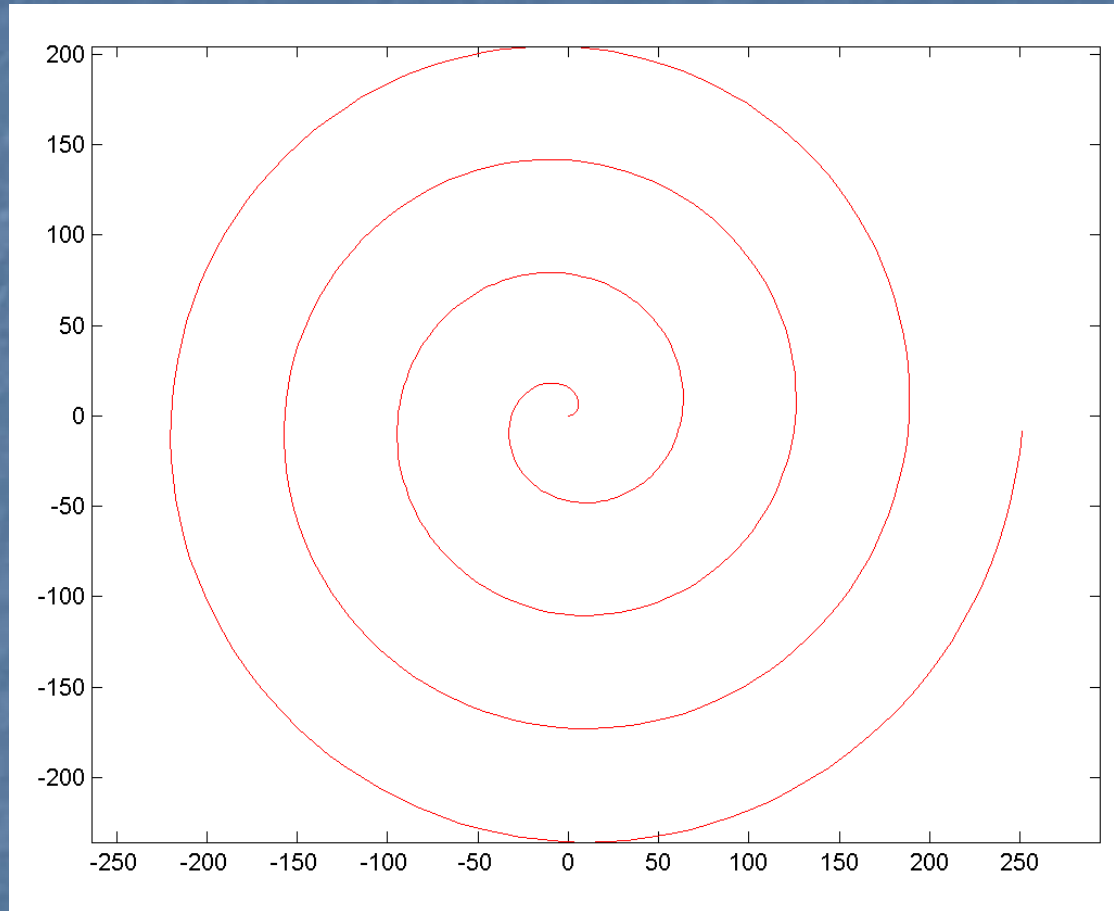


Decorative elements lying on radii of regular 32-gon.

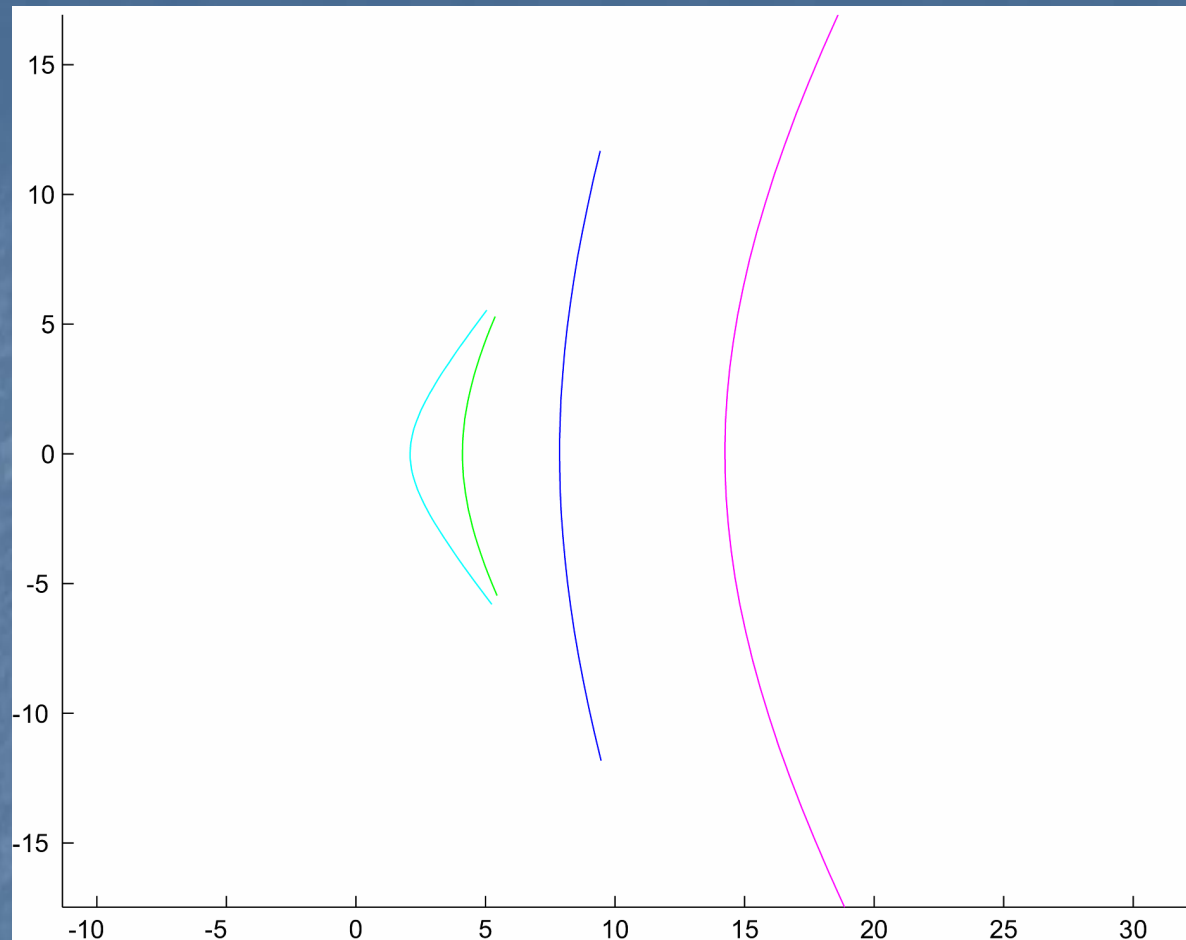
Geometric prototypes

The stability of the contour lines of numerous figures, appearing in various wall-paintings not associated with geometric themes, suggested the idea that these figures had probably been drawn by means of geometric stencils too.

Geometric prototypes



In the subsequent figures, **all line segments** shown **in red** are parts of the determined **Archimedes spiral**



In the subsequent figures, line segments' colors bi-univocally correspond to hyperbola stencils depicted above

$a=14.24, b=20.12$ (magenta),	$a=4.11, b=6.29$ (green),
$a=7.86, b=17.63$ (blue),	$a=2.09, b=2.52$ (cyan)

Some of the processed wall-paintings

The **color of each curve in (b) strictly corresponds to the same geometric prototype**. Once more the quality of the approximation is excellent



(a)



(b)

Geometric prototypes

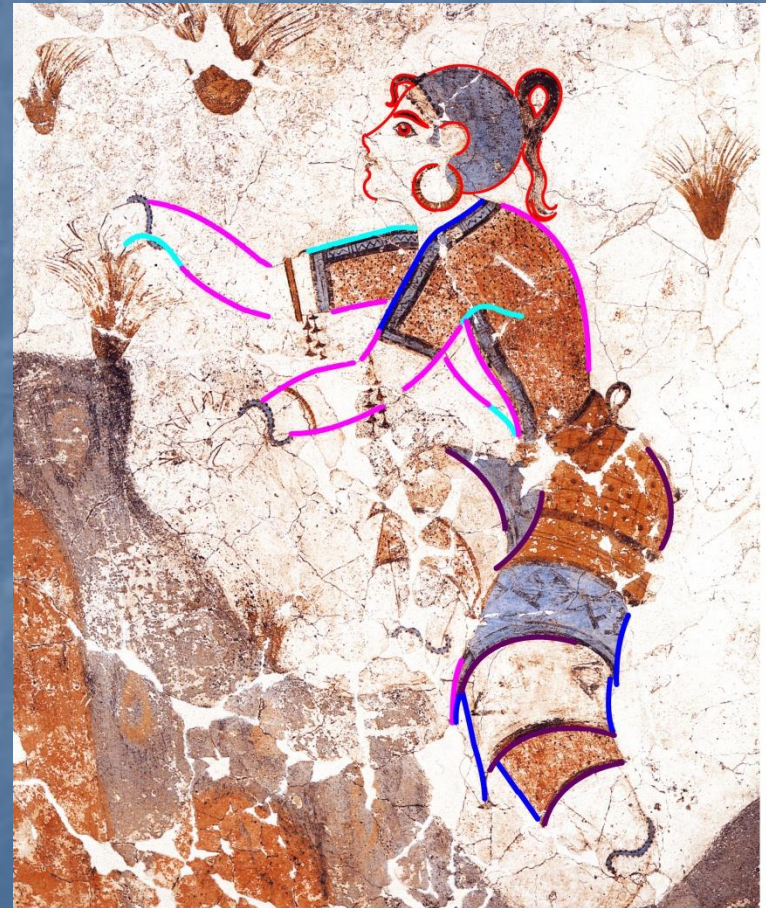
- The **entire head** of the lady had most probably been drawn via the use of a **single Archimedes spiral**.



All major figures of the wall-painting “**Gathering of crocus**” had most probably been dawn by means of the aforementioned stencils/guides

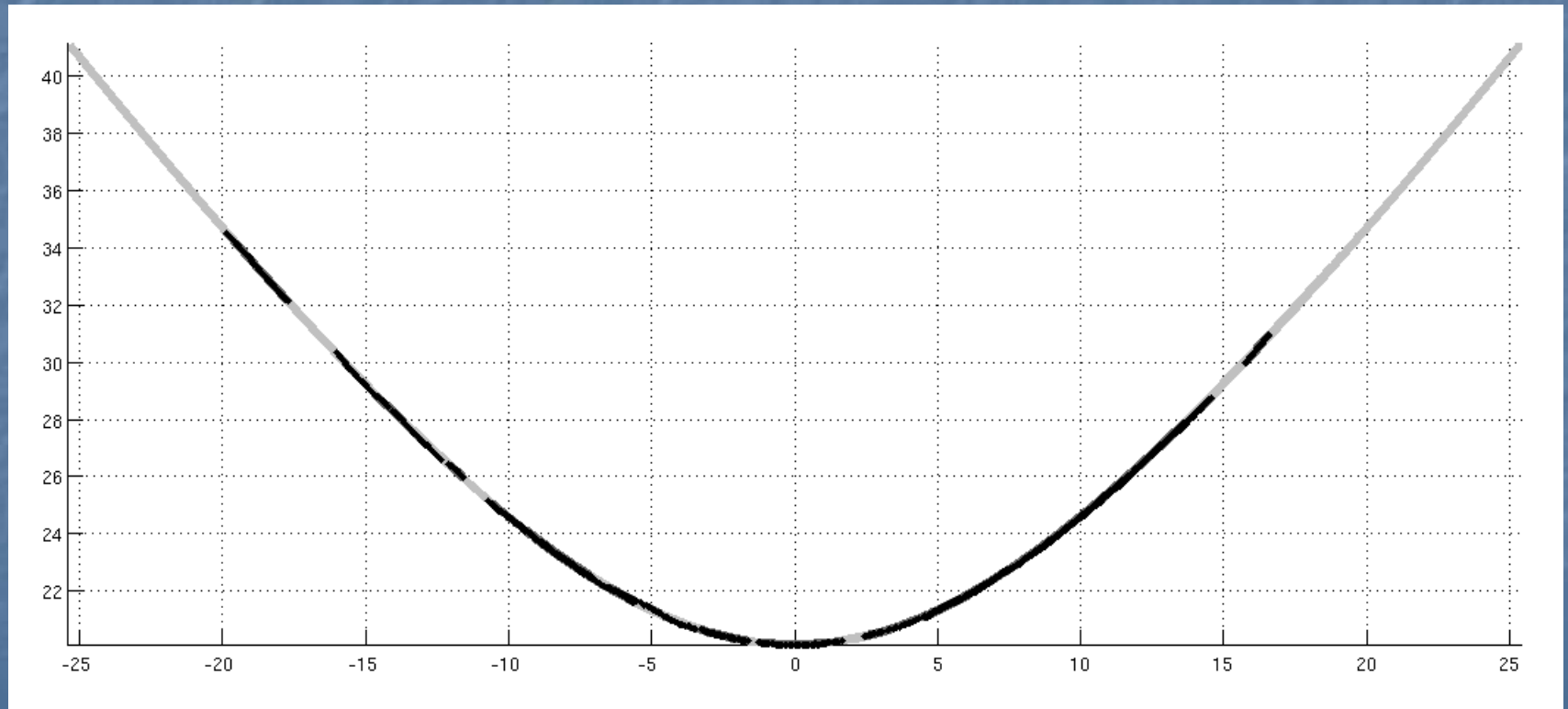


(a)

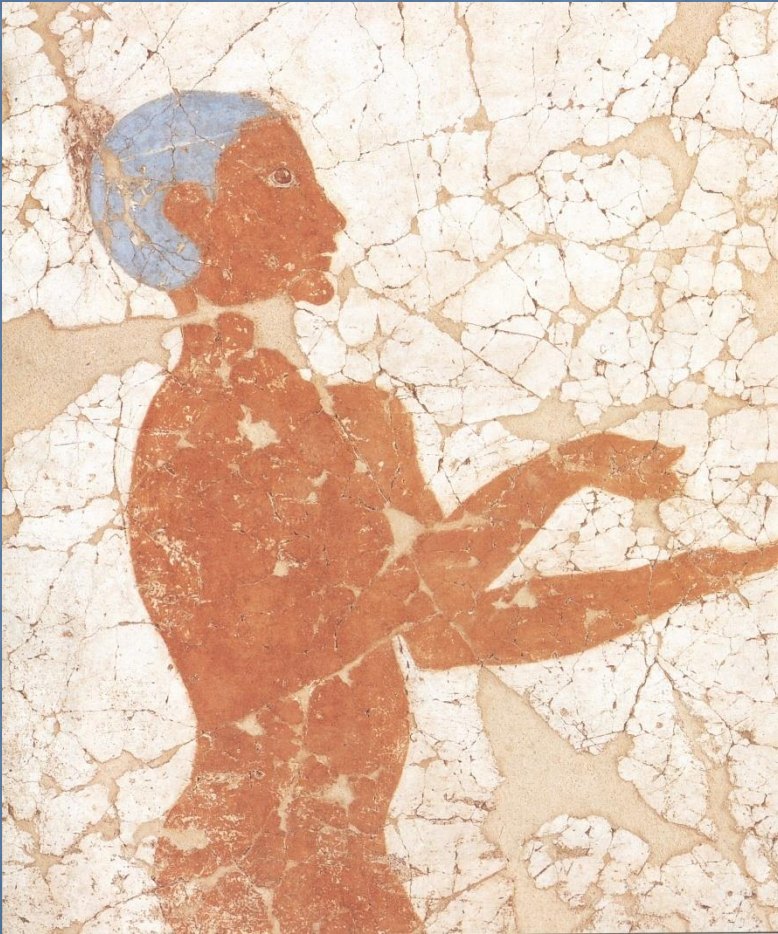


(b)

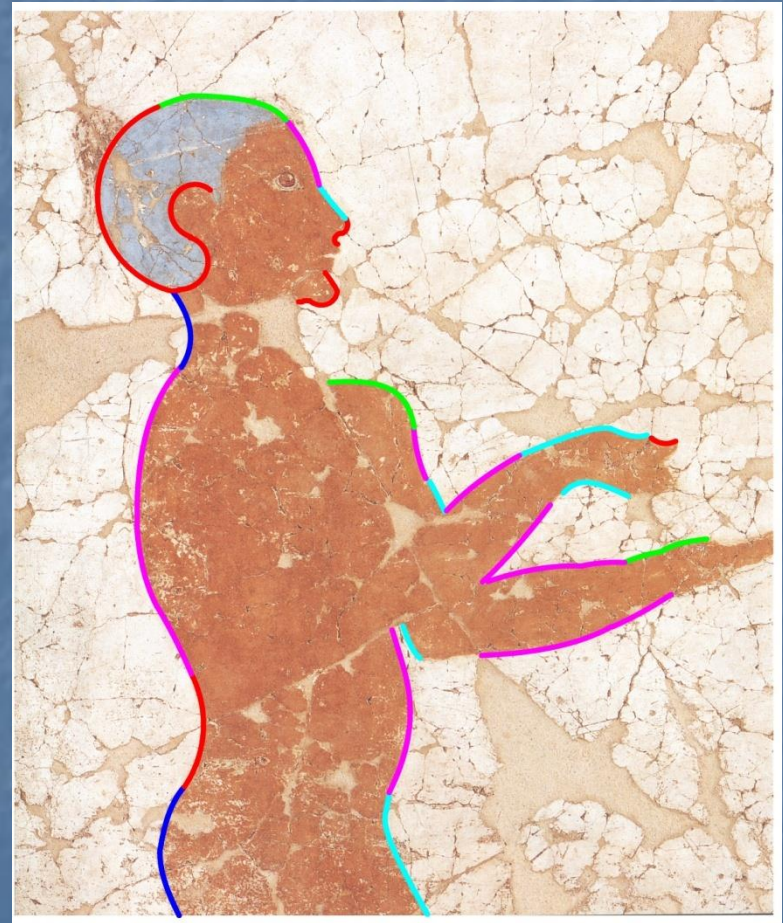
Drawn Outlines – to – Hyperbola prototype correspondence



All major figures of the wall-painting “**Naked boys**” had most probably been drawn by means of the aforementioned stencils/guides

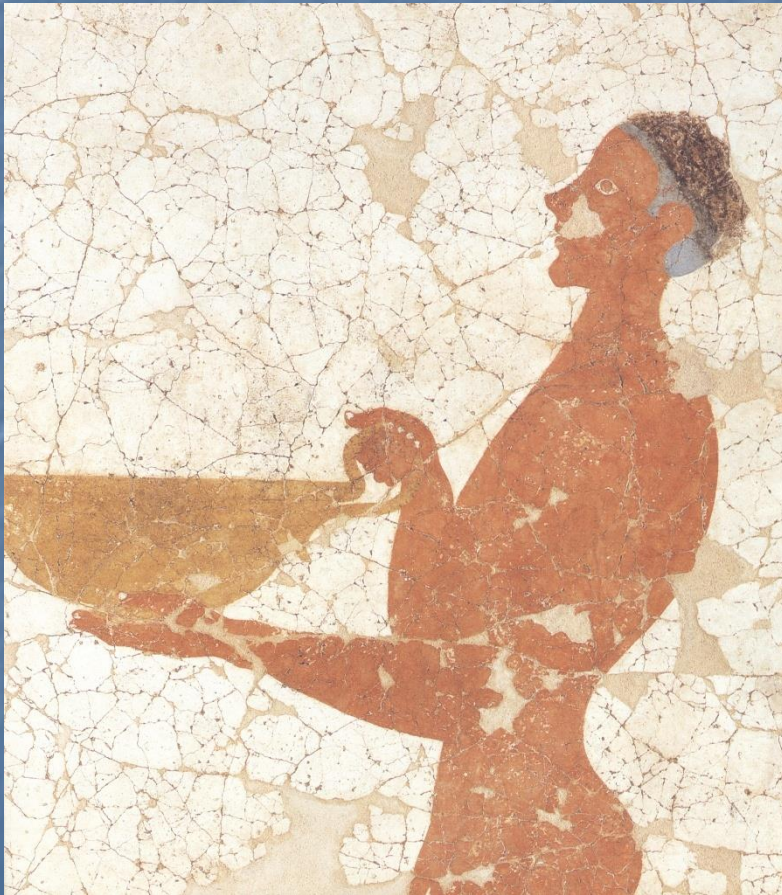


(a)



(b)

All major figures of the wall-painting “**Naked boys**” had most probably been dawn by means of the aforementioned stencils/guides



(a)

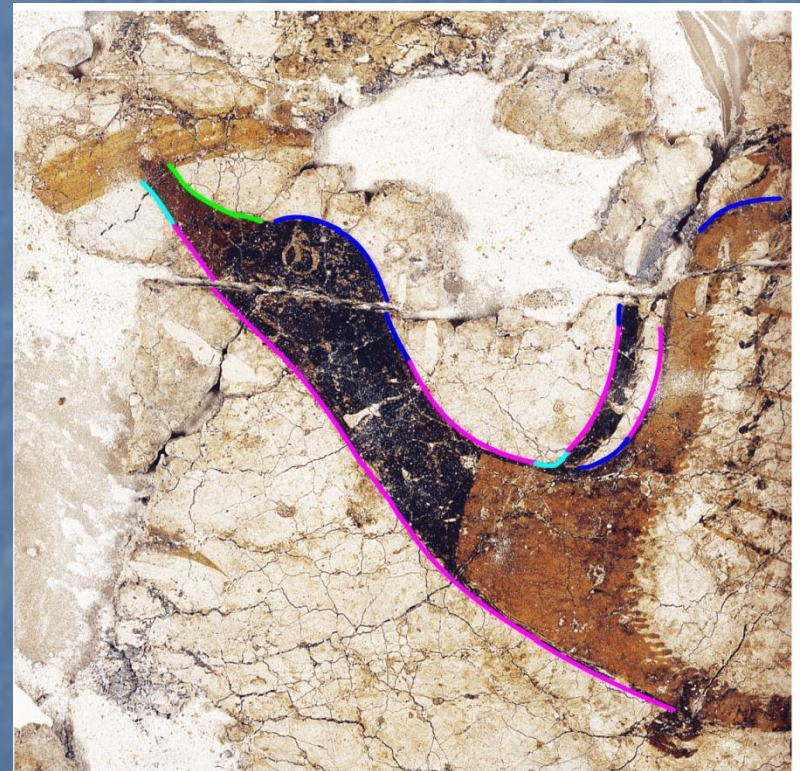


(b)

The wall-painting “**Wild duck**” had most probably been dawn by means of the aforementioned stencils/guides



(a)



(b)

“The Lady of Mycenae” was made by means of guides/stencils corresponding to **analogous geometric prototypes**

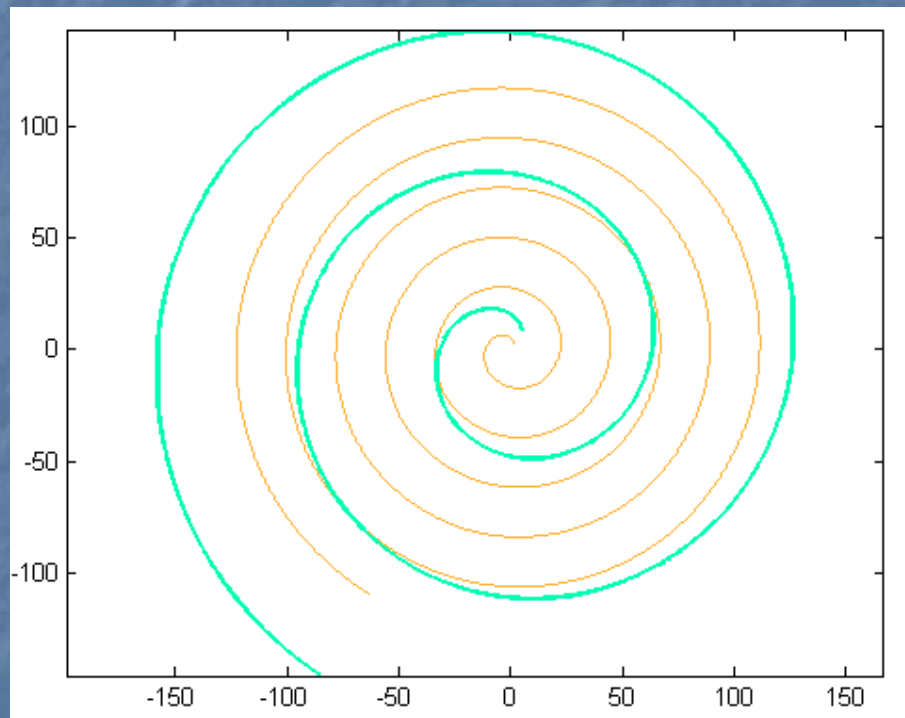
The outlines of “The Lady of Mycenae” correspond to **2 hyperbolae** and **2 linear spirals** in an excellent price-wise manner.

These guides belong to the **same class of curves** with the ones employed in **Thera**, but with **different parameters**.

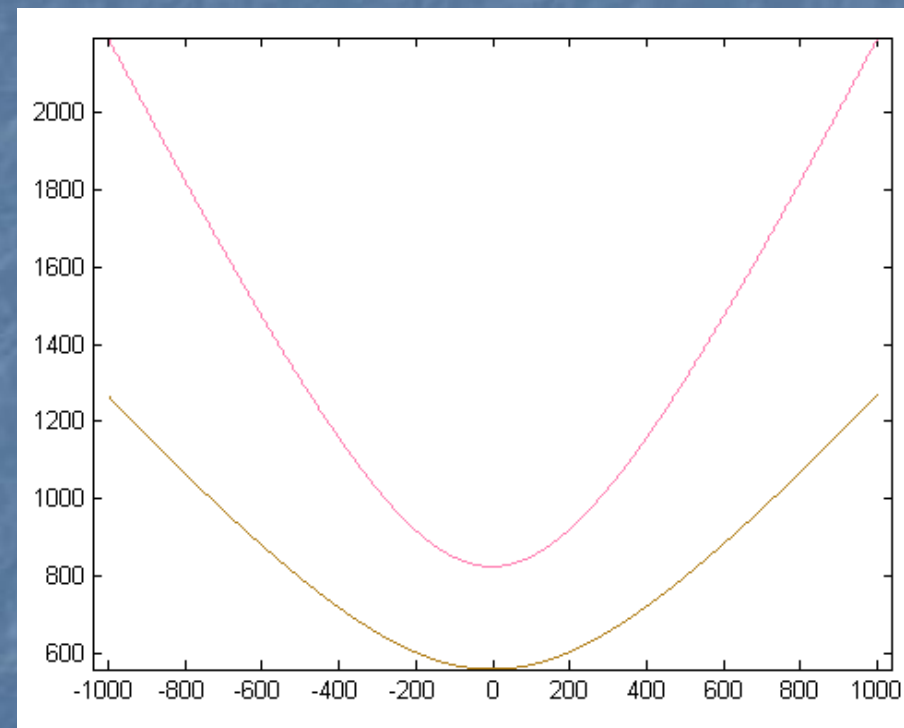
The Lady of Mycenae



Stencils most probably used for the drawing of "The Lady of Mycenae"

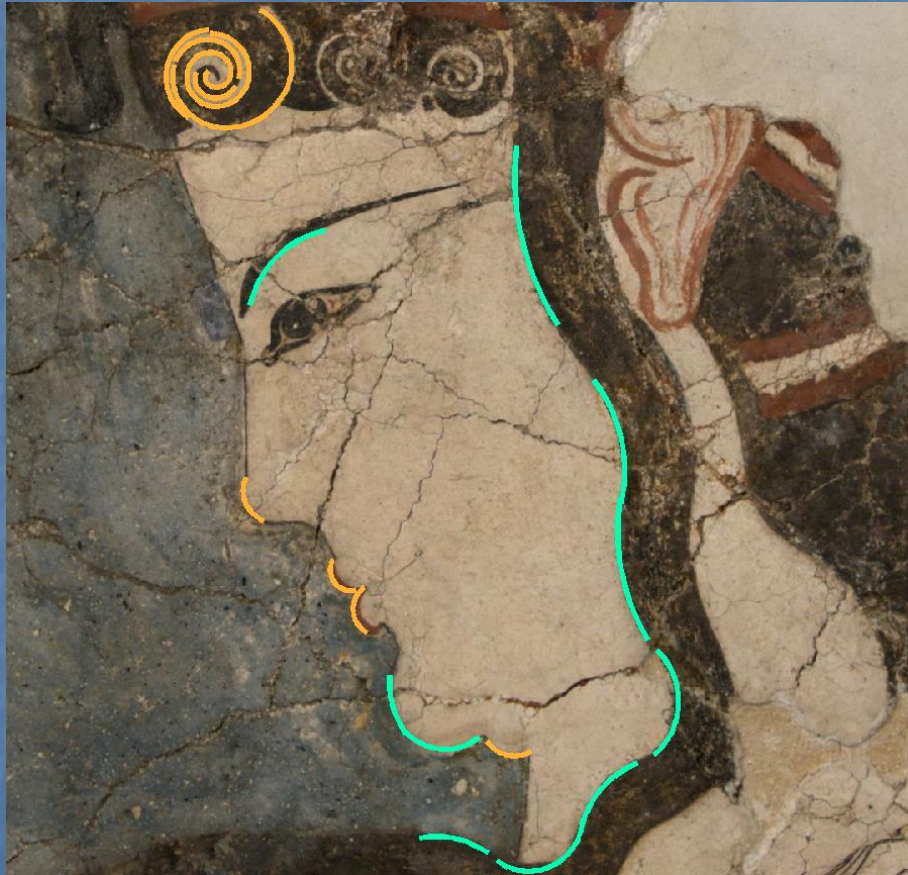


(a)



(b)

Spiral-stencils' parts probably used for drawing figure's face and hair outlines



(a)



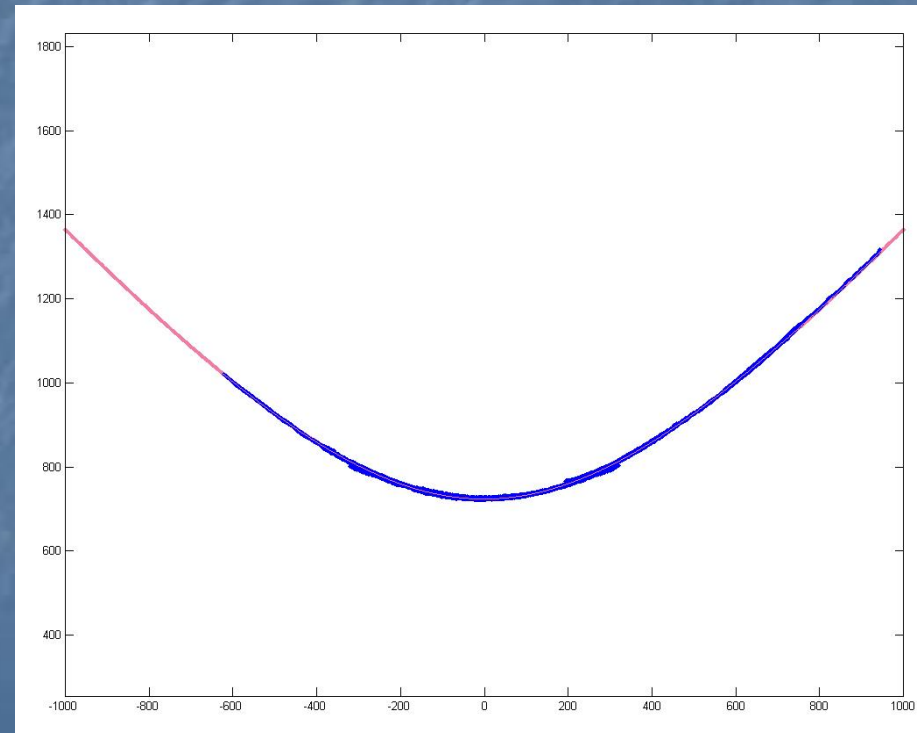
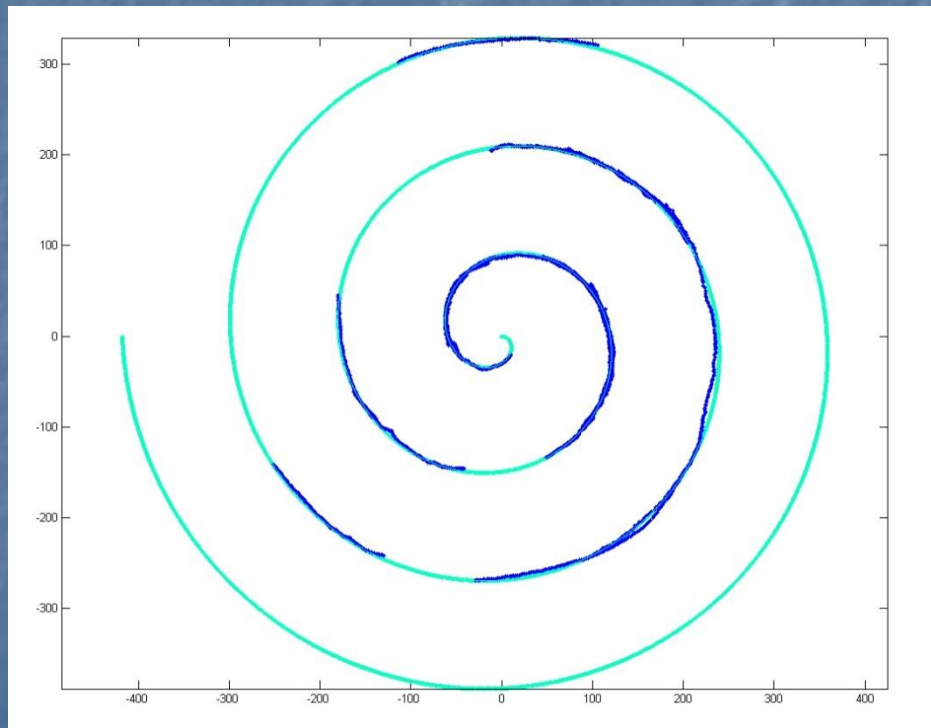
(b)

Hyperbolae and Spiral stencils' parts probably
used for drawing figure's body outlines



(c)

Drawn Parts – to – Prototypes correspondence



Conclusion

Realizations of Archimedes spirals and conics most probably appear c. 1350 years before their mathematic definition and treatment by Archimedes, Konon, Menaechmos, Apollonious, etc.

Conclusion

The sense of symmetry and geometry is prior to the strict mathematic formulation, is associated with Art and everyday life needs and it actually is responsible for the birth of mathematics and science.

Related publications in international journals

- C. Papaodysseus, Th. Panagopoulos, M. Exarhos, D. Fragoulis, G. Roussopoulos, P. Rousopoulos, G. Galanopoulos, C. Triantafyllou, A. Vlachopoulos, C. Doulas: "Distinct, late bronze age (c. 1650 B.C.) wall paintings of Akrotiri, Thera, comprising advanced geometrical patterns" Archaeometry, Oxford University, Blackwell Publishing, September 2005.
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- C. Papaodysseus, M. Exarhos, M. Panagopoulos, P. Rousopoulos, C. Triantafillou, Th. Panagopoulos "IMAGE AND PATTERN ANALYSIS FOR 1650 B.C.WALL PAINTINGS STUDY AND RECONSTRUCTION", IEEE Transactions on Systems Man and Cybernetics, Part A: Systems and Humans, Volume: 38, Issue: 4, July 2008.
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1. «**Ελευθεροτυπία**», 'Είχαν οι αρχαίοι στένσιλ;'
2. "**The Nature**" Were ancient Minoans centuries ahead of their time?
Unprecedented mathematical knowledge found in Bronze Age wall paintings.
3. "**Spectrum**" Geometrie ,Archimedische Vorläufer ,Geometrische Kenntnisse der Minoer
4. "**Frankfurter Allgemeine Zeitung**" " $r = a \varphi$, *aber hallo!*"
5. "**Republica**" **ROMA** "Il giallo dela Spirale di Archimede, A Creta era nota gia da mille anni. "
6. **The Manila Times**, "Were ancient Minoans centuries ahead of their time?"
7. **KOMPAS.com**, "Bangsa Minoa Telah Mengenal Ilmu Geometri?"
8. **Minerva magazine**, "Tracing the art of Akrotiri"

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